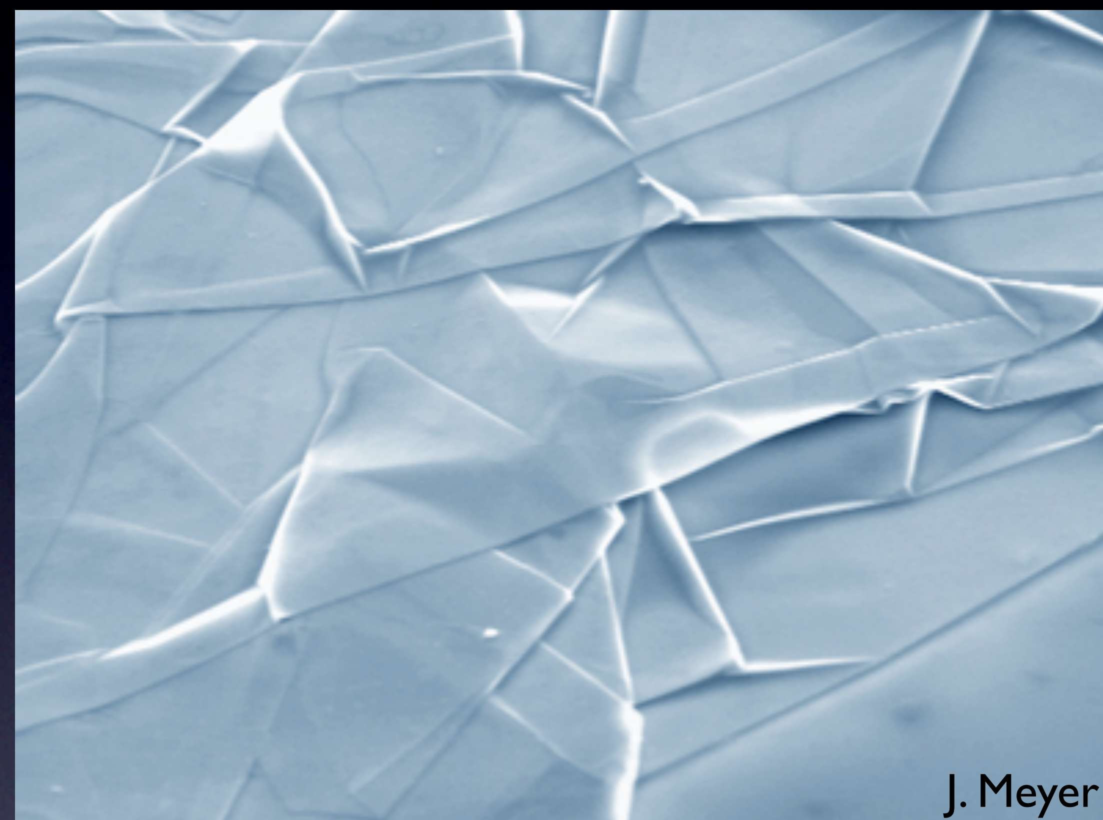
# The physics of Graphene




Eros Mariani

Freie Universität Berlin

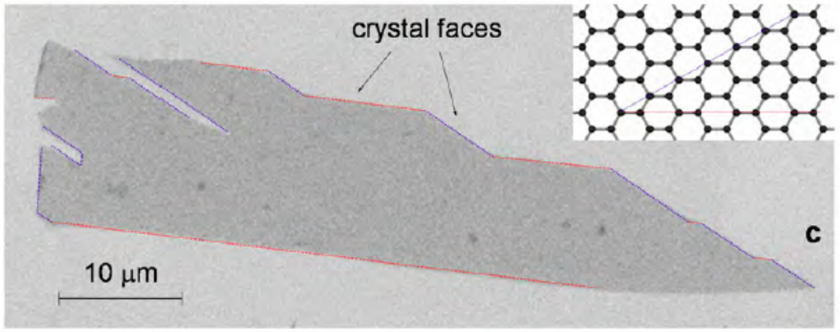
Freie Universität Berlin

# What is Graphene?

Single 2D sheet of carbon

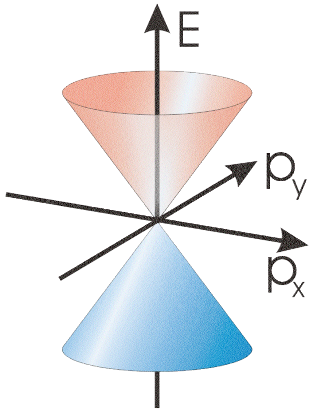
Does it exist?

First stable 2D crystal




A. Geim

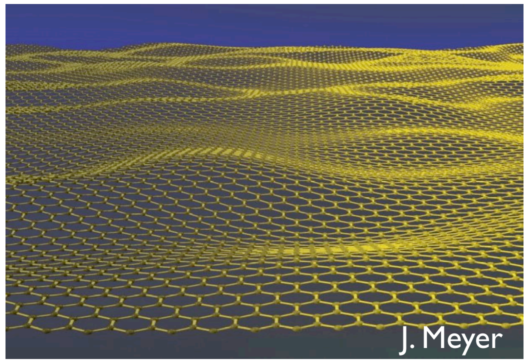
First massless Dirac fermions




News from Graphene!

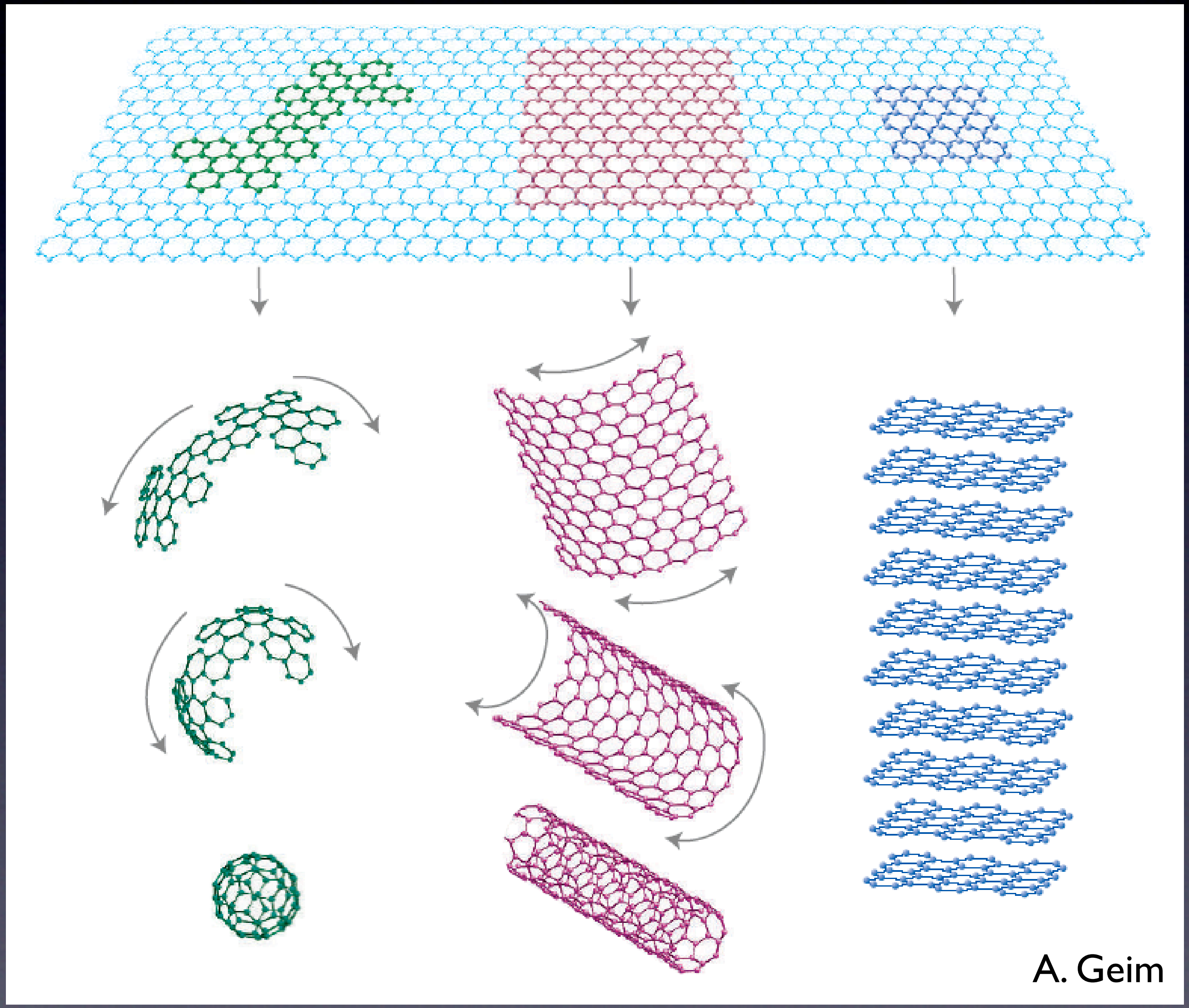
Interface between hard and soft condensed matter with QED

2D membrane in 3D space



# What is Graphene?

Single 2D sheet of carbon: one atom thick!




Graphene
2D

Graphene as the “mother” of 0D, 1D and 3D carbon allotropes

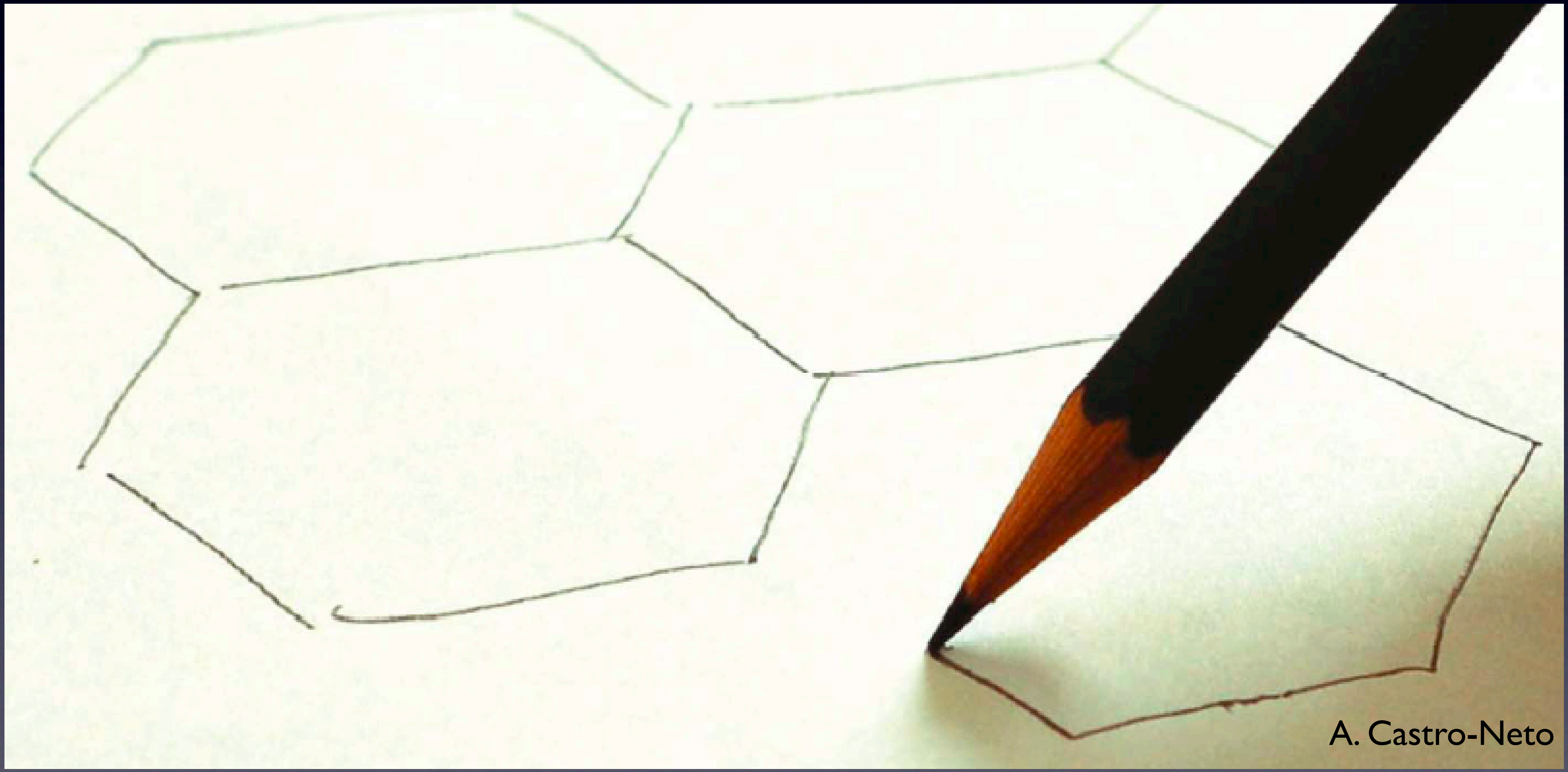
Fullerene
0D

Carbon nanotubes
1D

Graphite
3D

# How to obtain graphene?

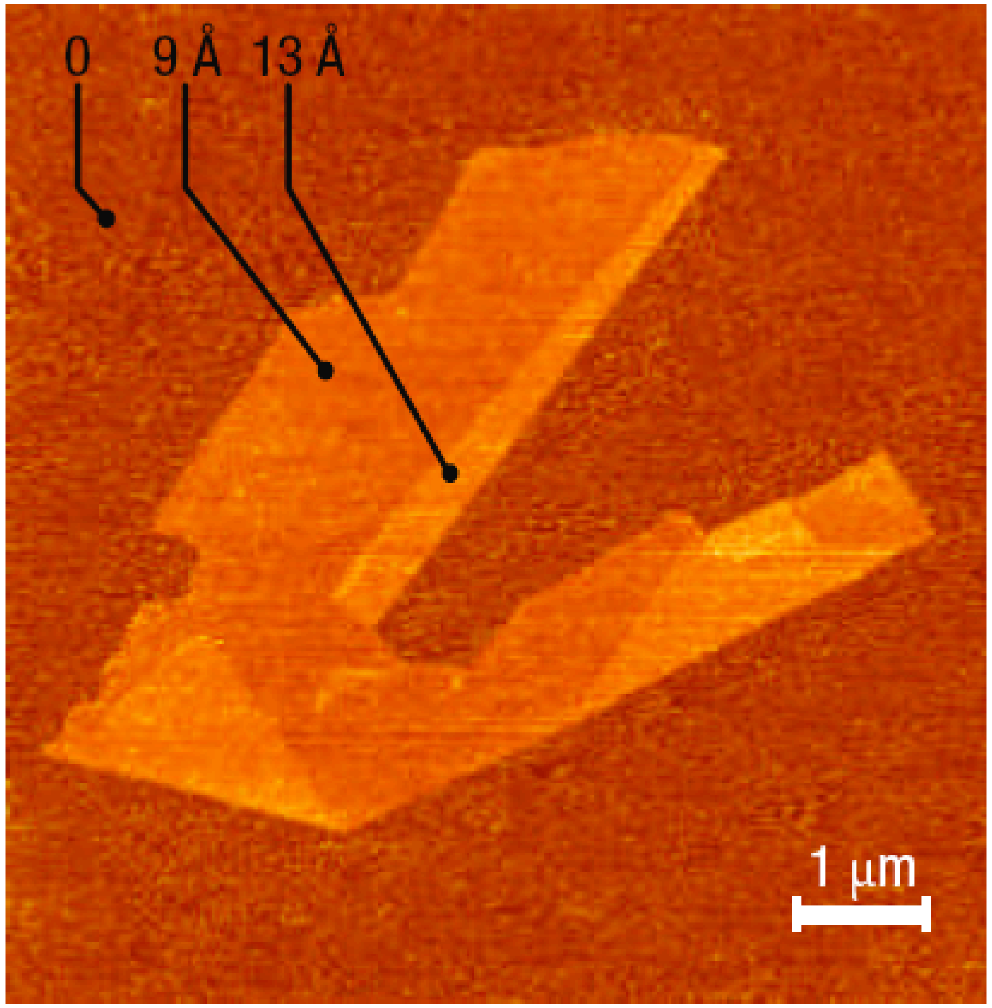
## Experiment Live !!!

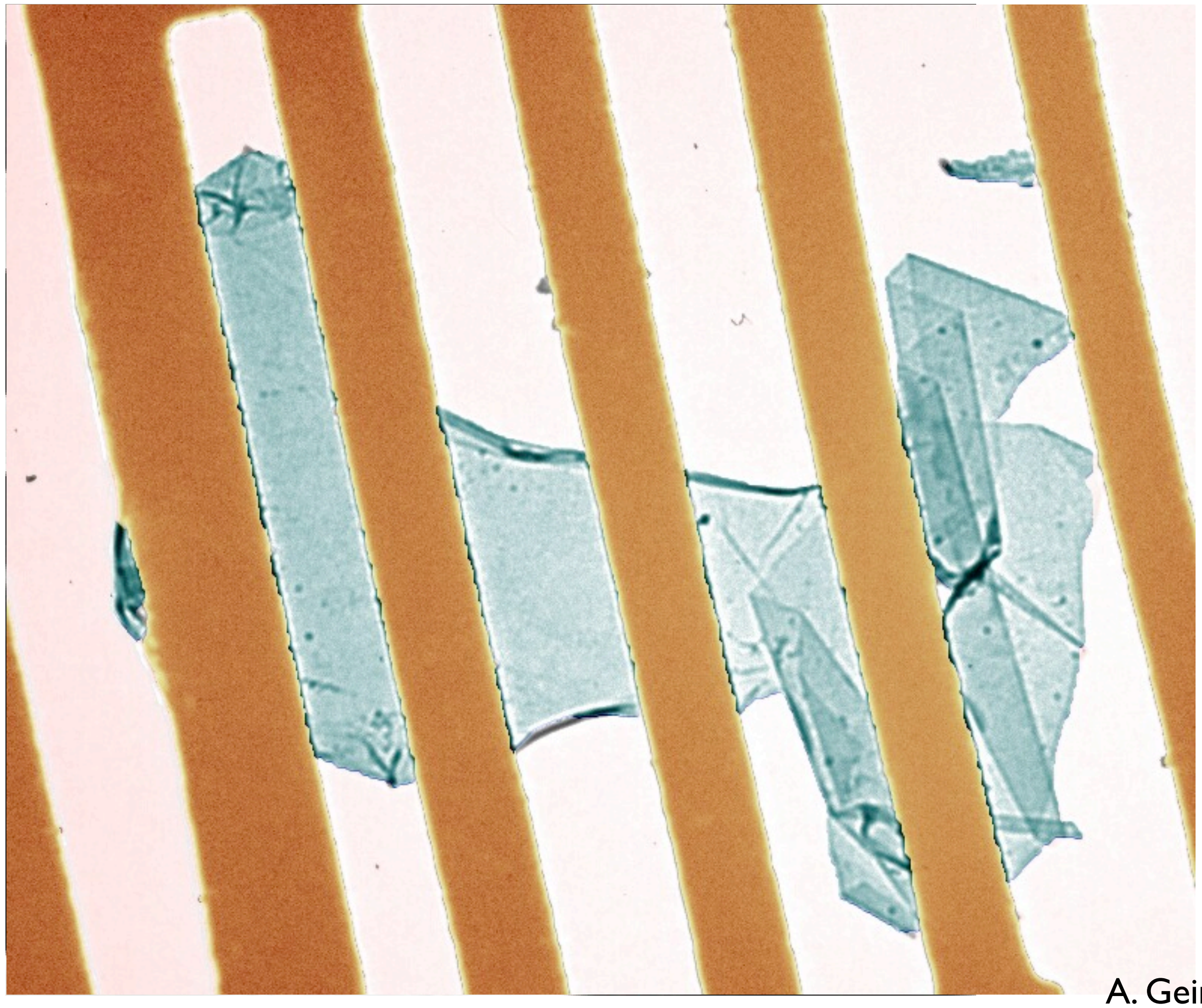



Novoselov et al., Nature 2005
Zhang et al., Nature 2005

# Real graphene flakes

Novoselov et al., Nature 2005
Zhang et al., Nature 2005

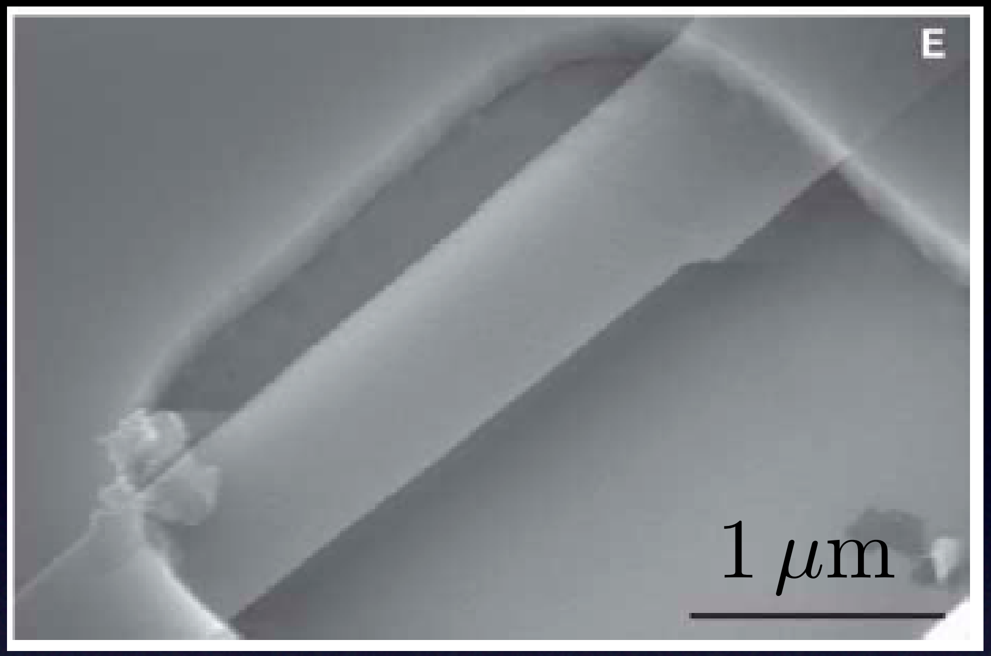


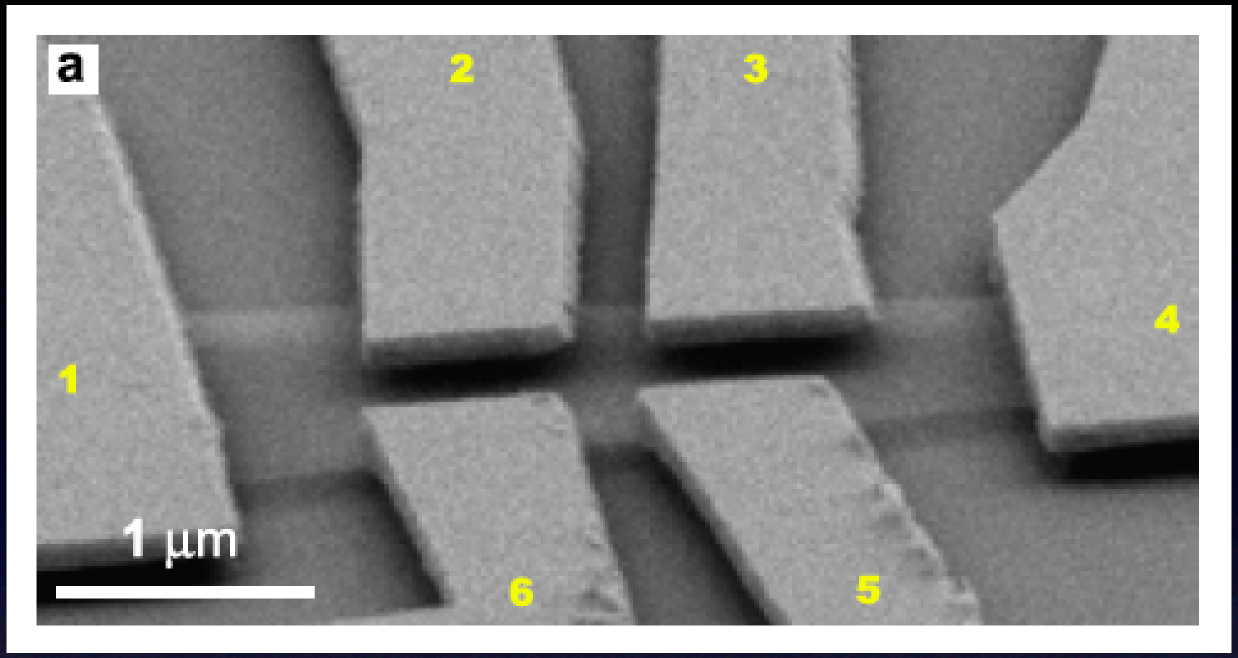
A. Geim

Geim group, Manchester
Kim group, Columbia (NY)

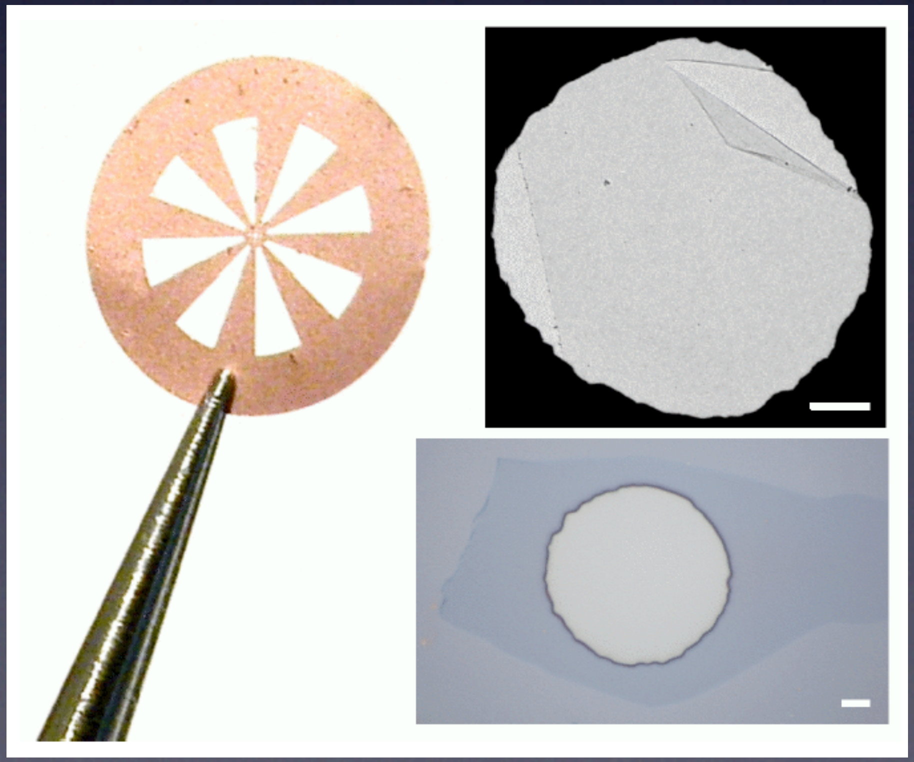
# Further graphene flakes




McEuen group, Science 07
Geim group, Nature 07




Kim group 08



Geim group 08

# Would you like a graphene sheet?

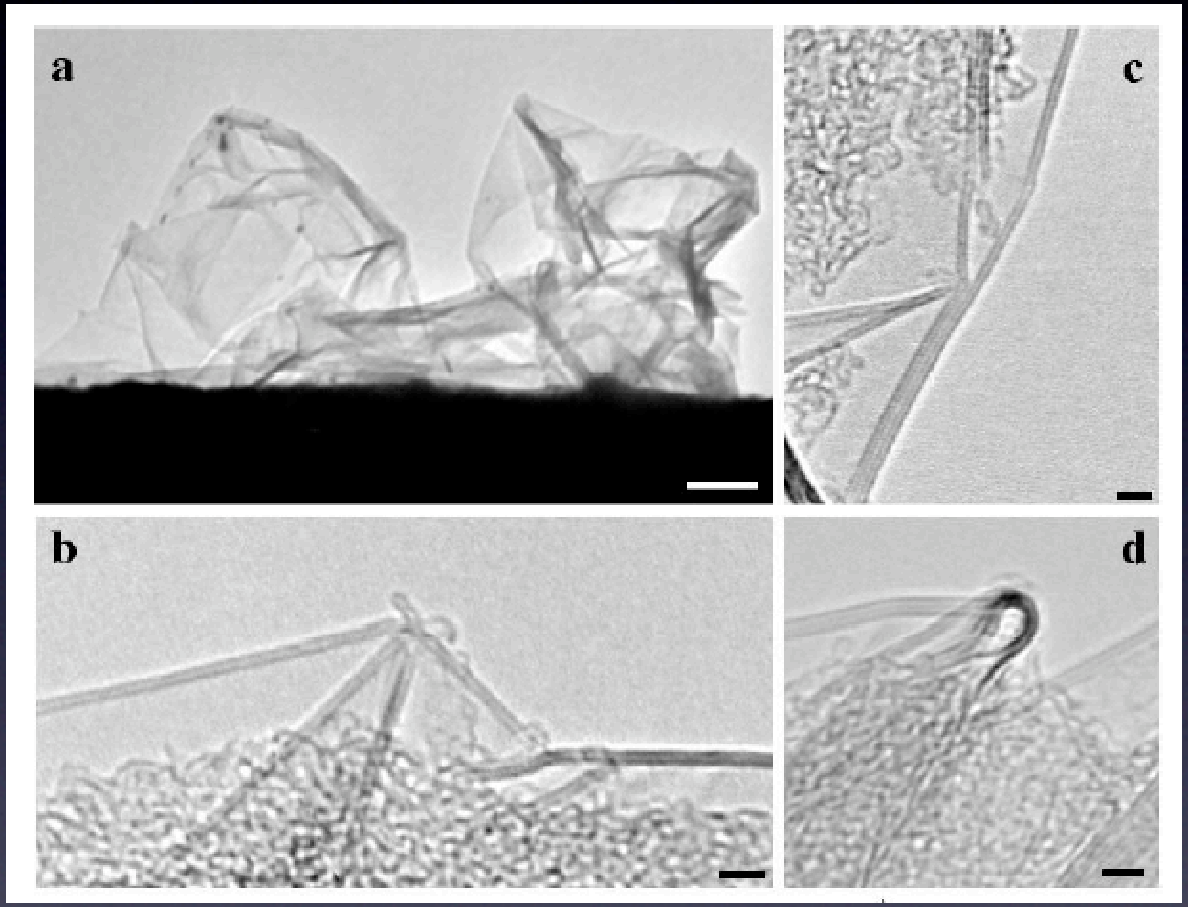
**Sample: S936**

| | |
|---|---|
| **Flake Area ($\mu m^2$) ± 10%** | 1300 |
| **Substrate** | 300nm $SiO_2$ on n-doped 380μm Si |
| **Price** | £400 |

from: www.grapheneindustries.com

# Q: Do 2D crystals exist?

Peierls
Landau
Mermin-Wagner
Kosterlitz-Thouless



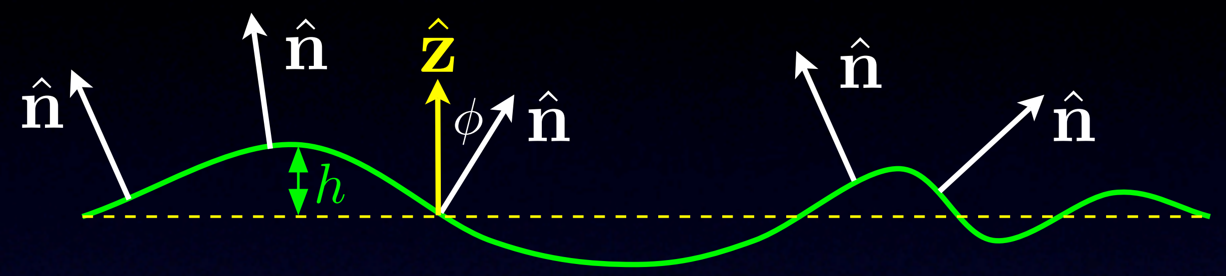
A. Geim

Flakes tend to crumple

# Problems with harmonic theory

(Nelson & Peliti 87)

$$F_{\text{bend}} = \frac{1}{2}\int \mathrm{d}\mathbf{r}\,\kappa(\nabla^2 h)^2$$




$$\langle \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \rangle = \langle \cos\phi \rangle \simeq 1 - \frac{1}{2}\langle \phi^2 \rangle$$

# Problems with harmonic theory

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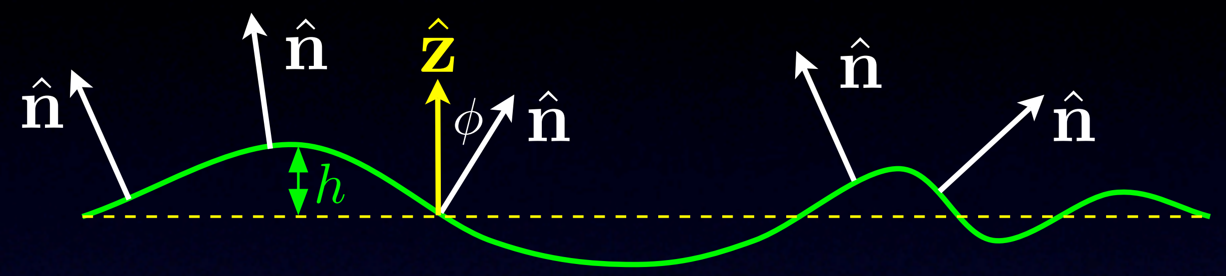



$$\langle \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \rangle = \langle \cos\phi \rangle \simeq 1 - \frac{1}{2}\langle \phi^2 \rangle$$

$$\langle \phi^2(\mathbf{r}) \rangle \sim \langle |\nabla h(\mathbf{r})|^2 \rangle = \int \mathrm{d}\mathbf{q}\, q^2 \frac{T}{\kappa q^4}$$

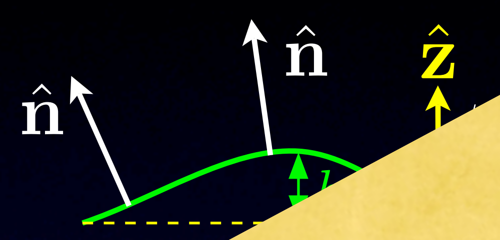
logarithmically singular at **any** finite T

The membrane would be unstable against crumpling!!!

# Problems with harmonic theory

(Nelson & Peliti 87)

$F_{h} \ldots h)^2$



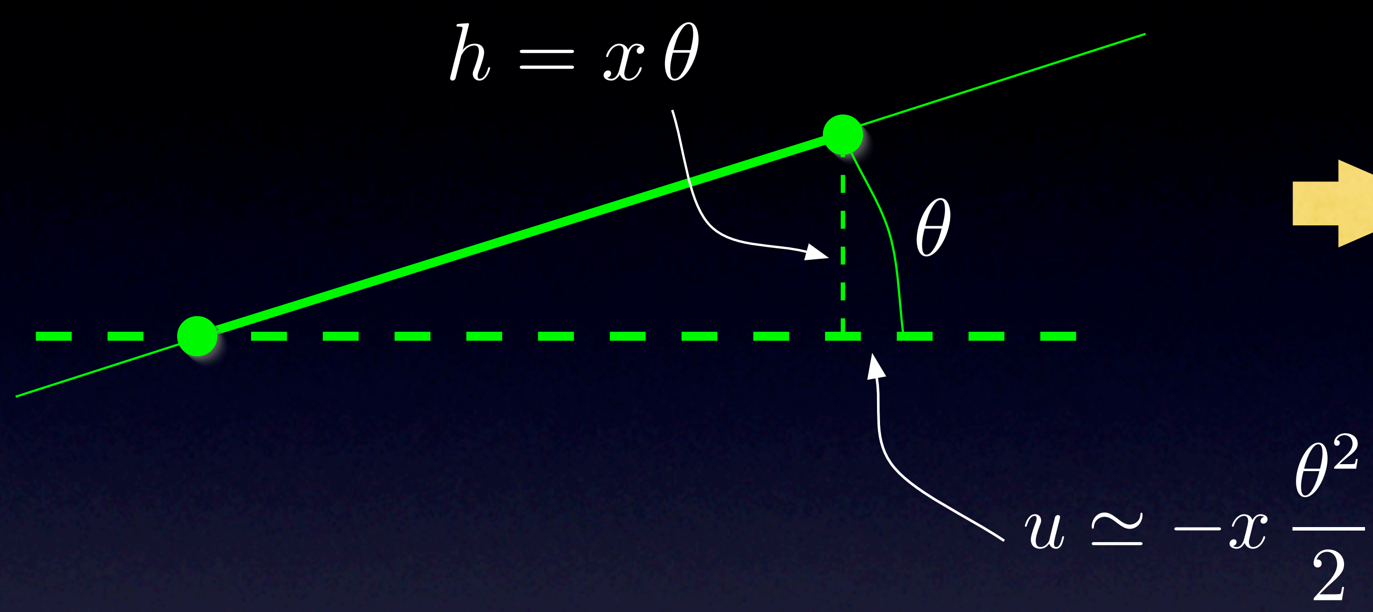

BUT…

$\kappa q^4$

logarithmically singular at **any** finite T

Th… would be unstable against crumpling!!!

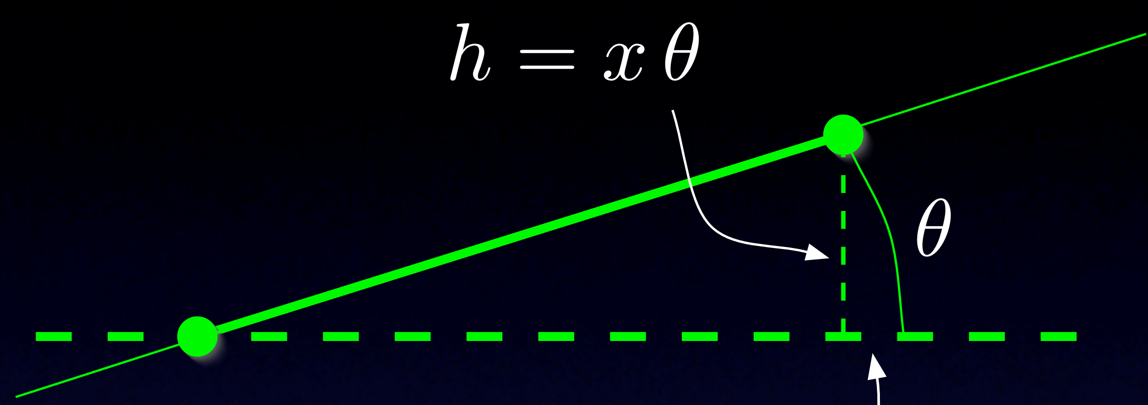
# Coupling between bending and stretching




Vanishing strain

$$u_{ij} = 0$$

# Coupling between bending and stretching



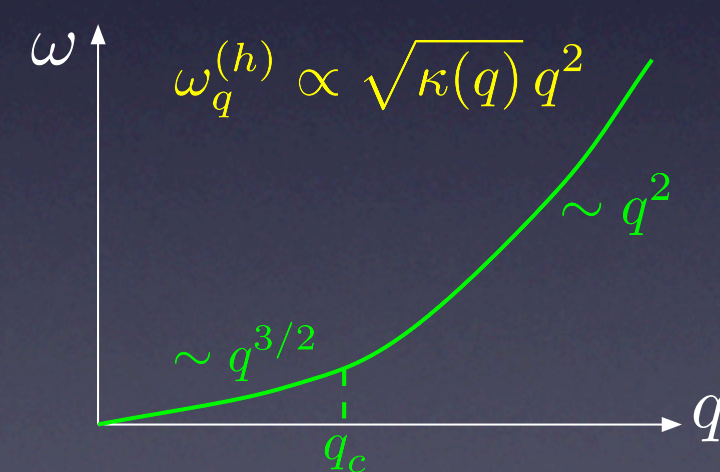

$u \simeq -x\,\frac{\theta^2}{2}$

Vanishing strain

$\Rightarrow \quad u_{ij} = 0$

$$\boxed{\kappa(q) = \kappa\sqrt{1 + q_c^2/q^2}}$$

$$q_c = \sqrt{\frac{3K_0 k_{\mathrm{B}} T}{8\pi\kappa^2}}$$

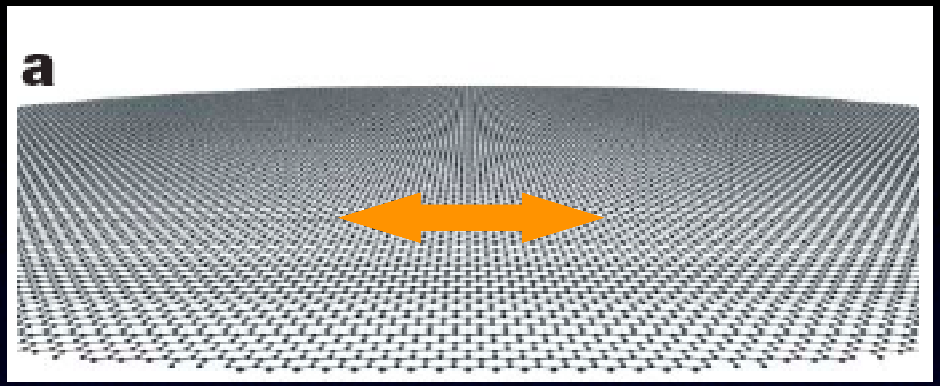



Singularity removed: Stable flat phase !

# “New” Phonons in graphene

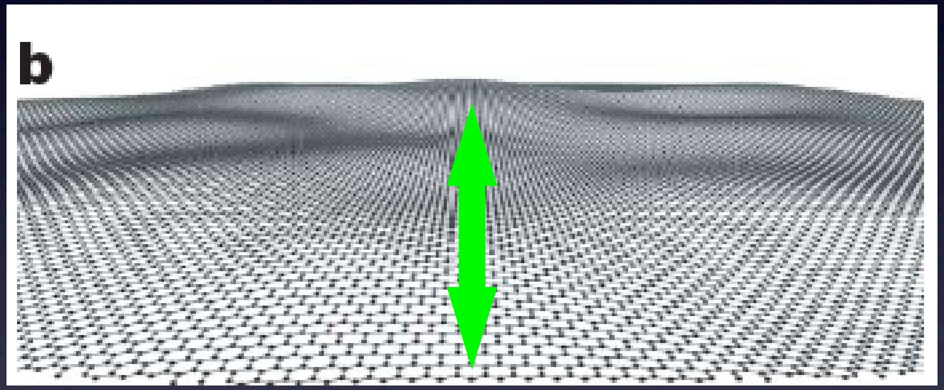
Woods ’00
Katsnelson ‘07
Castro-Neto ‘07

In-plane modes




Hard to excite
Good coupling to electrons

Flexural modes



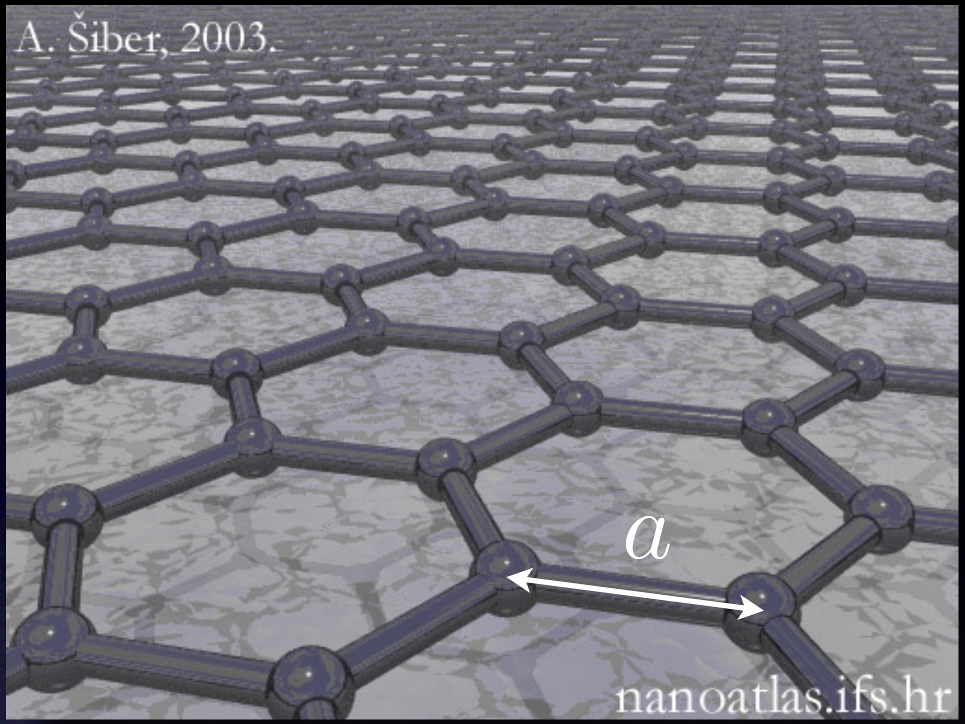

Soft to excite
Weak coupling to electrons

At low T flexural modes dominate the resistivity yielding $\rho \propto T^{5/2} \ln T$ vs. the usual $T^4$ for in-plane modes

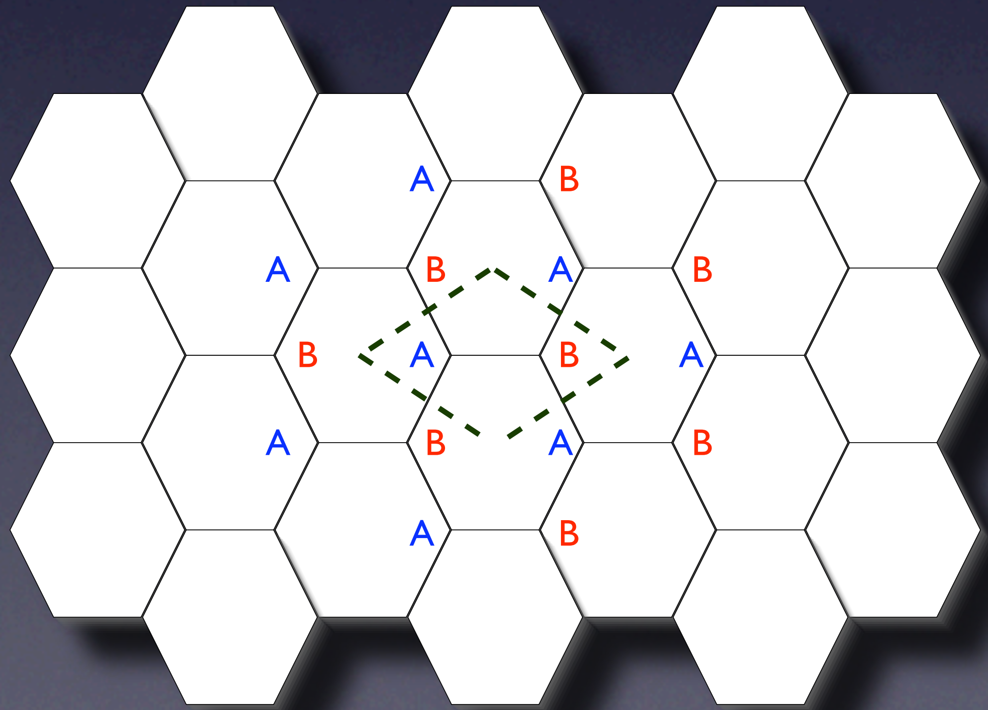
E. Mariani and F. von Oppen, PRL 2008

# Electronic properties of Graphene

**Graphene:**
single 2D sheet
of carbon atoms




$a = 1.42\ \text{Å}$




Two inequivalent sublattices A and B

Tight binding

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j$$

# Bloch bands in Graphene

(Wallace '47)

$$|\psi_{\mathbf{k}}\rangle = \sum_{A} u_{A,\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{R}_A} |\mathbf{R}_A\rangle + \sum_{B} u_{B,\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{R}_B} |\mathbf{R}_B\rangle$$

$$\begin{bmatrix} 0 & -t \sum_{j=1}^{3} e^{-i\mathbf{k}\cdot\mathbf{e}_j} \\ -t \sum_{j=1}^{3} e^{i\mathbf{k}\cdot\mathbf{e}_j} & 0 \end{bmatrix} \begin{bmatrix} u_{A,\mathbf{k}} \\ u_{B,\mathbf{k}} \end{bmatrix} = E_{\mathbf{k}} \begin{bmatrix} u_{A,\mathbf{k}} \\ u_{B,\mathbf{k}} \end{bmatrix}$$

# Bloch bands in Graphene

(Wallace '47)

$$|\psi_{\mathbf{k}}\rangle = \sum_A u_{A,\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{R}_A}\,|\mathbf{R}_A\rangle + \sum_B u_{B,\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{R}_B}\,|\mathbf{R}_B\rangle$$

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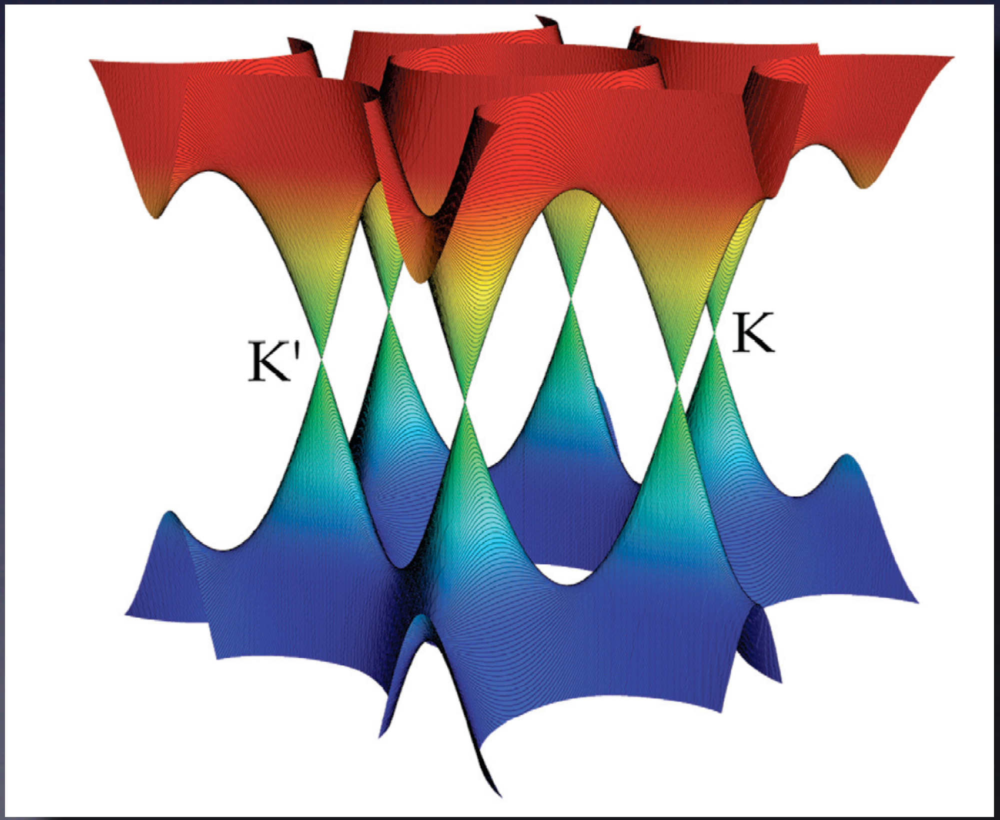
## Spectrum

$$E_{\mathbf{k}} = \pm t\sqrt{\left|\sum_{i=1}^{3} e^{i\mathbf{k}\cdot\hat{\mathbf{e}}_i}\right|^2}$$

Two inequivalent Dirac cones



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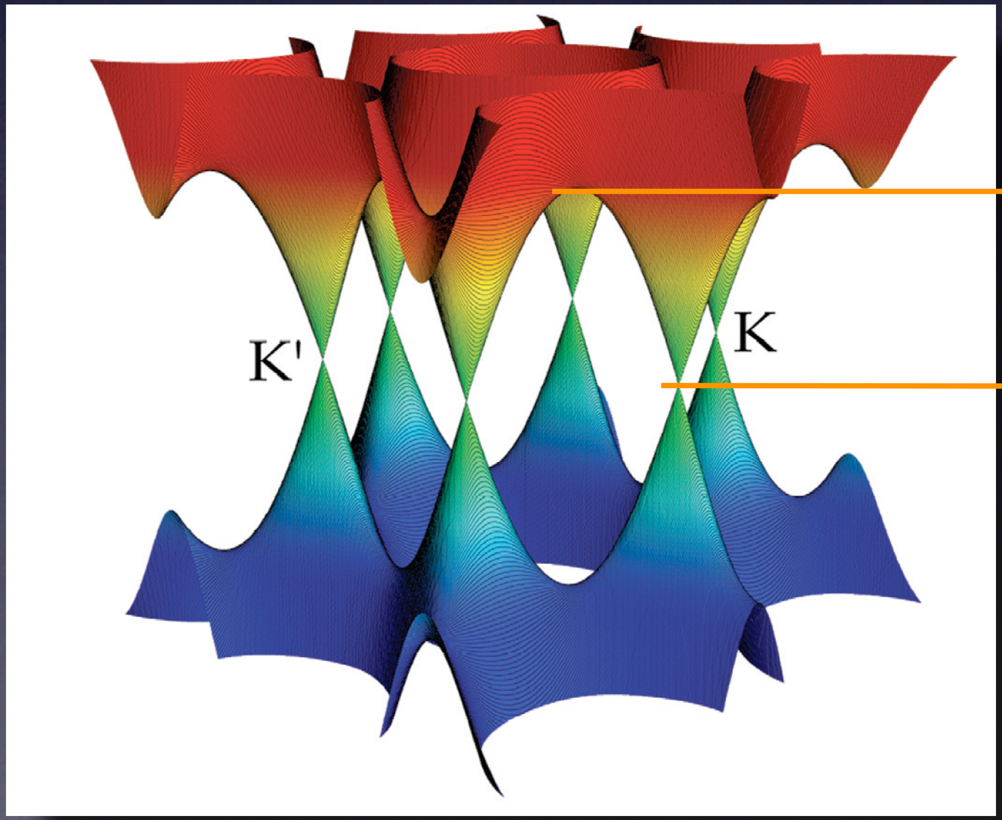
Spectrum

$$E_{\mathbf{k}} = \pm t \sqrt{\left| \sum_{i=1}^{3} e^{i\mathbf{k}\cdot\hat{\mathbf{e}}_i} \right|^2}$$

Two inequivalent
Dirac cones




$\simeq 10^5\ K$

# Low energy Hamiltonian

(in each cone)

$$H = v\,\vec{\sigma}\cdot\mathbf{p}$$

$$H = \mathrm{v}\begin{array}{c} \left[\begin{array}{cccc} 0 & p_x - ip_y & 0 & 0 \\ p_x + ip_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -(p_x - ip_y) \\ 0 & 0 & -(p_x + ip_y) & 0 \end{array}\right] \end{array} \begin{array}{c} A_+ \\ B_+ \\ B_- \\ A_- \end{array}$$

$$v \simeq \frac{c}{300}$$

Low energy massless Dirac spectrum

$$E = \pm vp$$

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(in each cone)

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$$v \simeq \frac{c}{300}$$

Low energy massless Dirac spectrum

$$E = \pm vp$$

Chirality

$$\vec{\sigma}\cdot\hat{\mathbf{p}} = \pm 1$$

Fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \rightarrow \frac{e^2}{\hbar v} \simeq 2$$

- ✓ Stable 2D crystals exist
- ✓ Massless Dirac fermions

Consequences of Chirality

Klein paradox

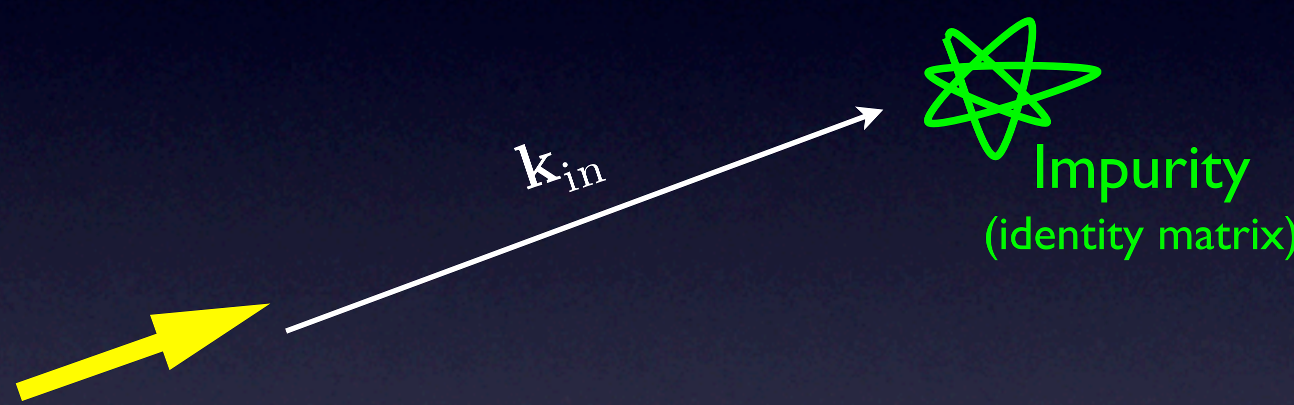
Inhomogeneities and interactions

# Chirality

absence of elastic backscattering

$\sigma \cdot \hat{\mathbf{k}} = +1$

$\sigma \cdot \hat{\mathbf{k}} = -1$



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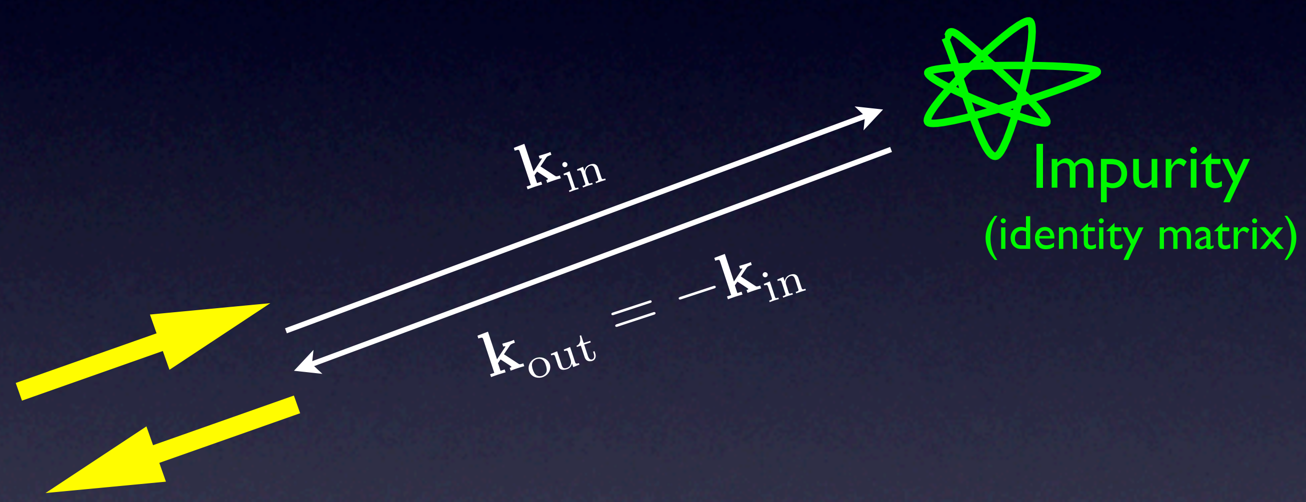


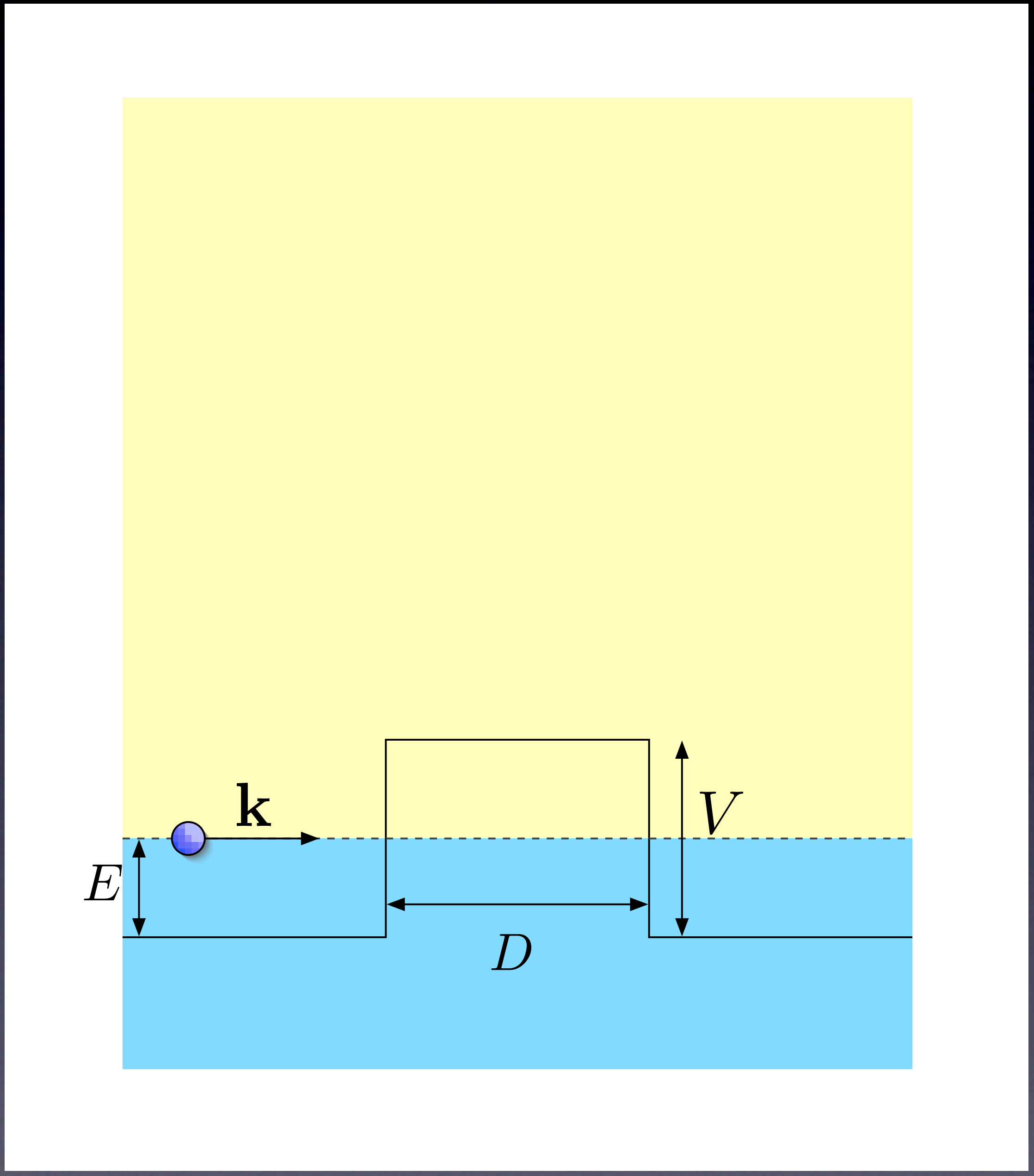

$$\langle \uparrow | \rangle \downarrow = 0$$

High mobility!

mean free path $l \simeq 1\,\mu\mathrm{m}$

# Klein paradox

Tunneling through
a high barrier



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Tunneling through
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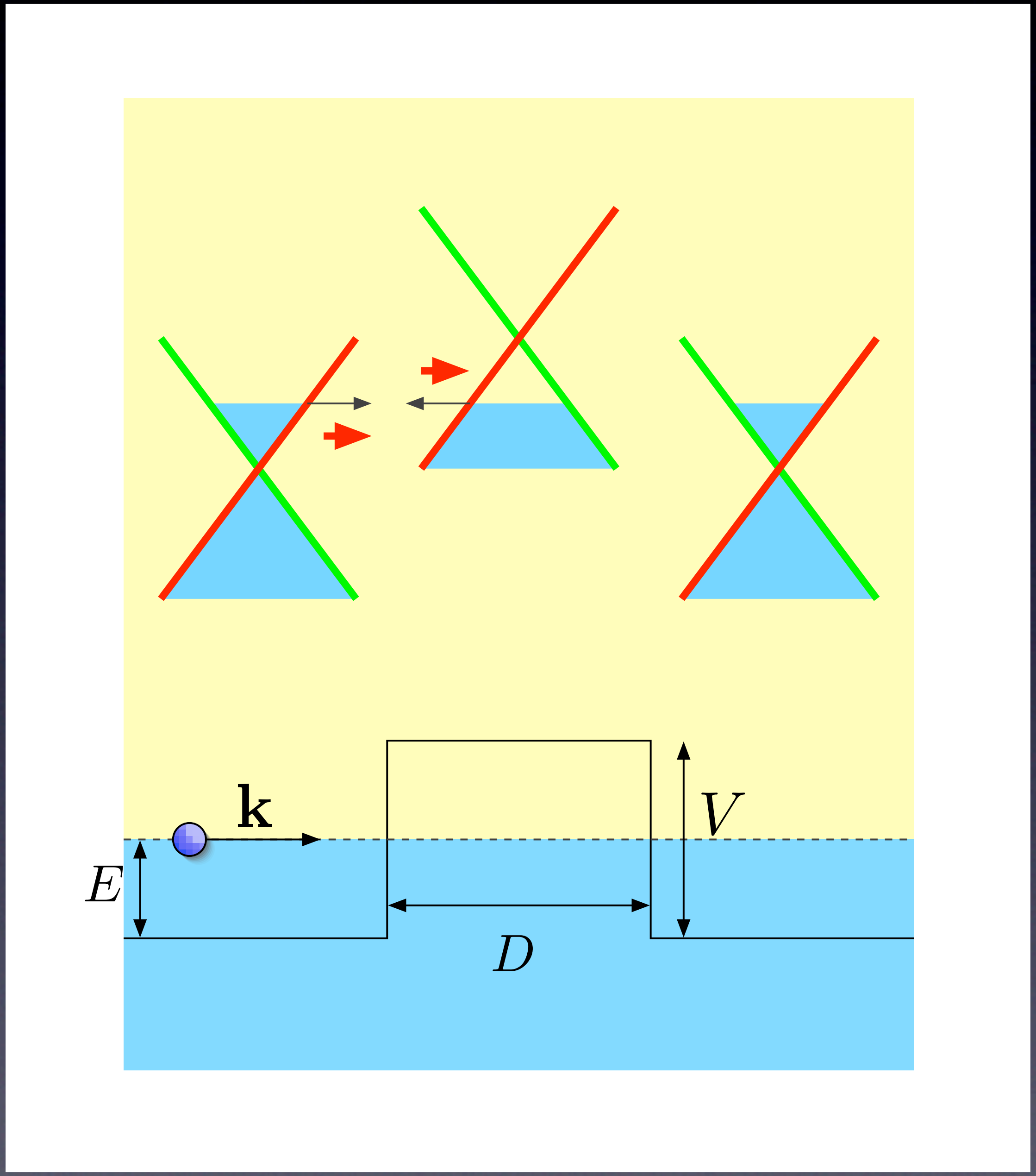



Transmission
probability

$$T \rightarrow 1$$

for

$$V \gg E$$

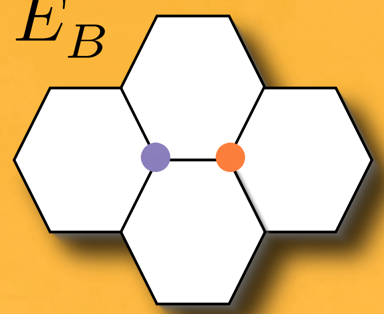
"Unstoppable"
electrons

# Inhomogeneities in the Dirac theory

Peres et al., PRB 2006
McCann et al., PRL 2006
Aleiner et al., PRL 2006
Altland, PRL 2006
Ostrovsky et al., 2006

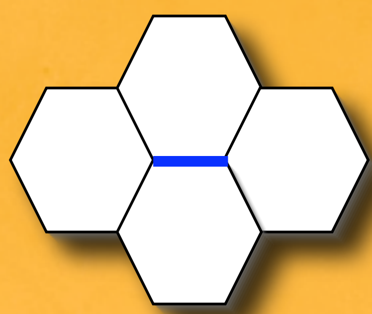
**On-site disorder**

$E_A \neq E_B$



**Hopping modulations**

(e.g. distortions)



$t_{AB} \neq t$

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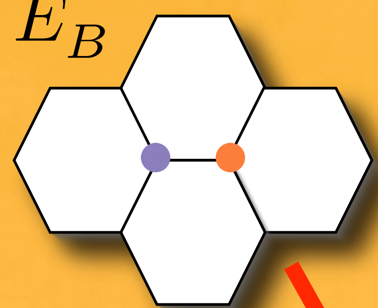

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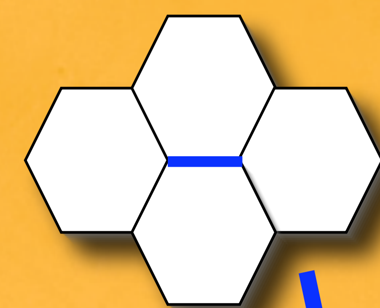


$t_{AB} \neq t$

$$H = \mathrm{v} \begin{array}{c} \left[ \begin{array}{cc} \delta t_{\mathrm{on-site}} & p_x - i p_y + \delta t_{\mathrm{hop}} \\ p_x + i p_y + \delta t_{\mathrm{hop}} & -\delta t_{\mathrm{on-site}} \end{array} \right] \end{array} \begin{array}{c} A \\ B \end{array}$$

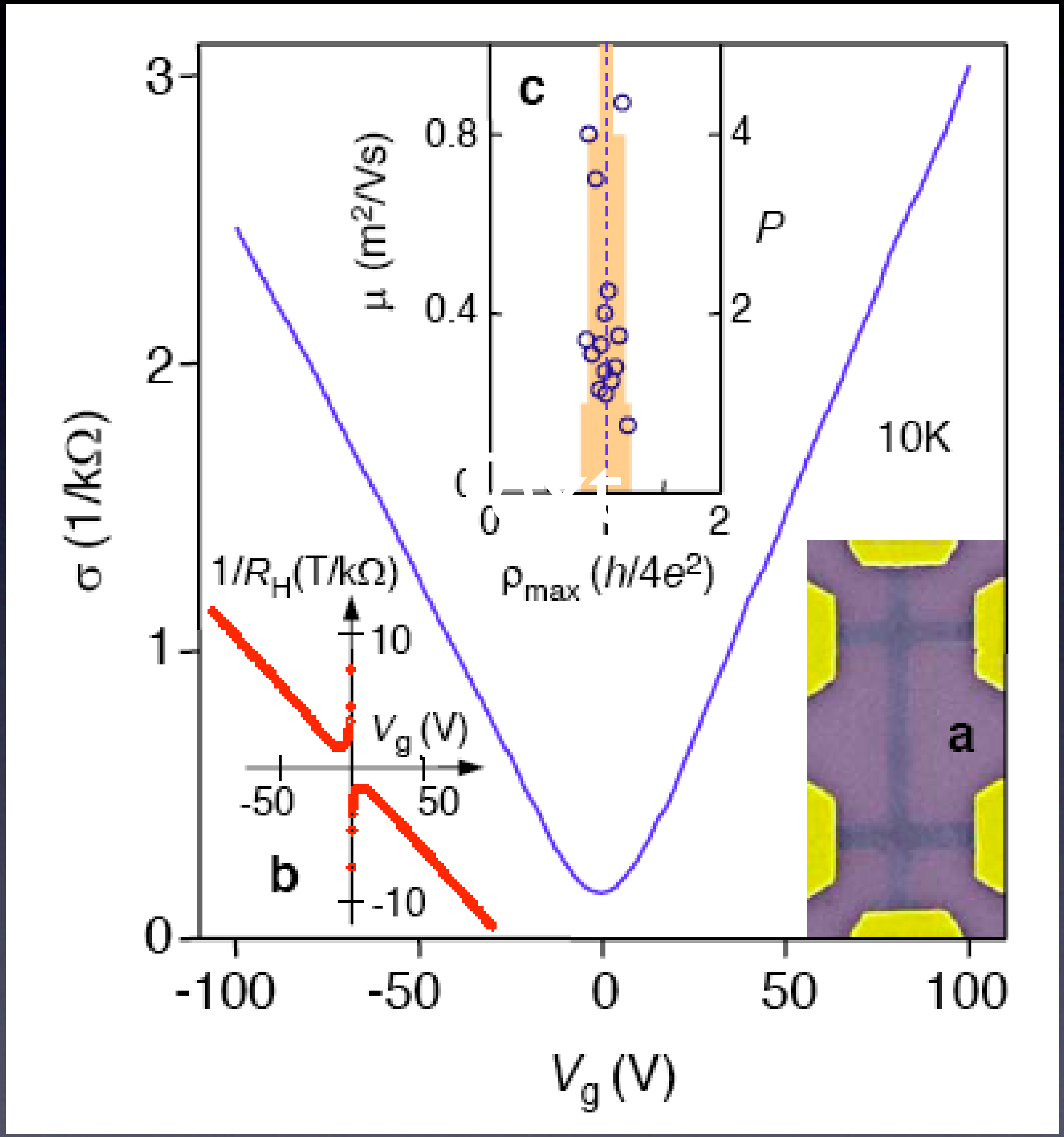
$$H = \Pi_z \otimes \sigma \cdot (\mathbf{p} - \frac{e}{c}\mathbf{A}) + V(\mathbf{r})$$

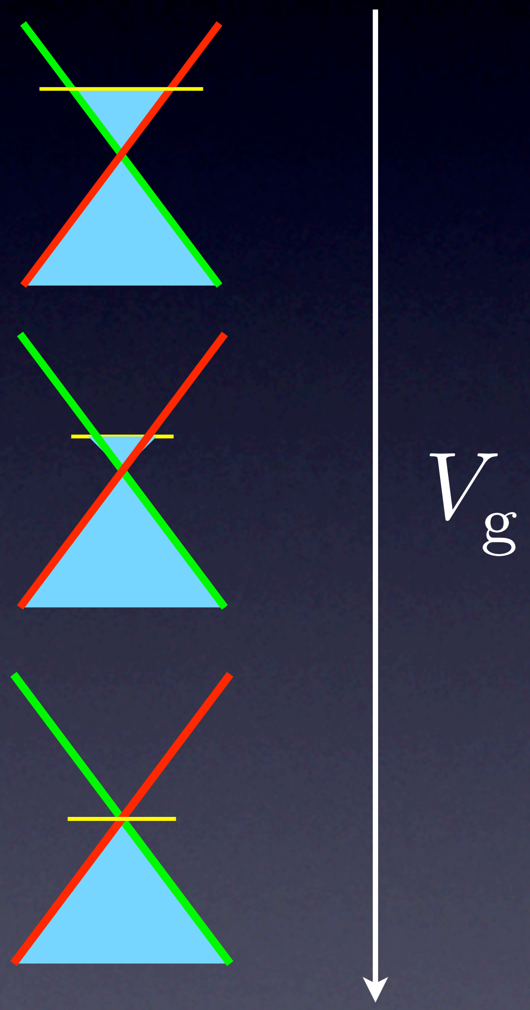
**Random potential** vs
**Random gauge field**

**Q: Signatures of their contributions?**

# Finite conductivity without carriers

Linear electric field effect: no localized charges

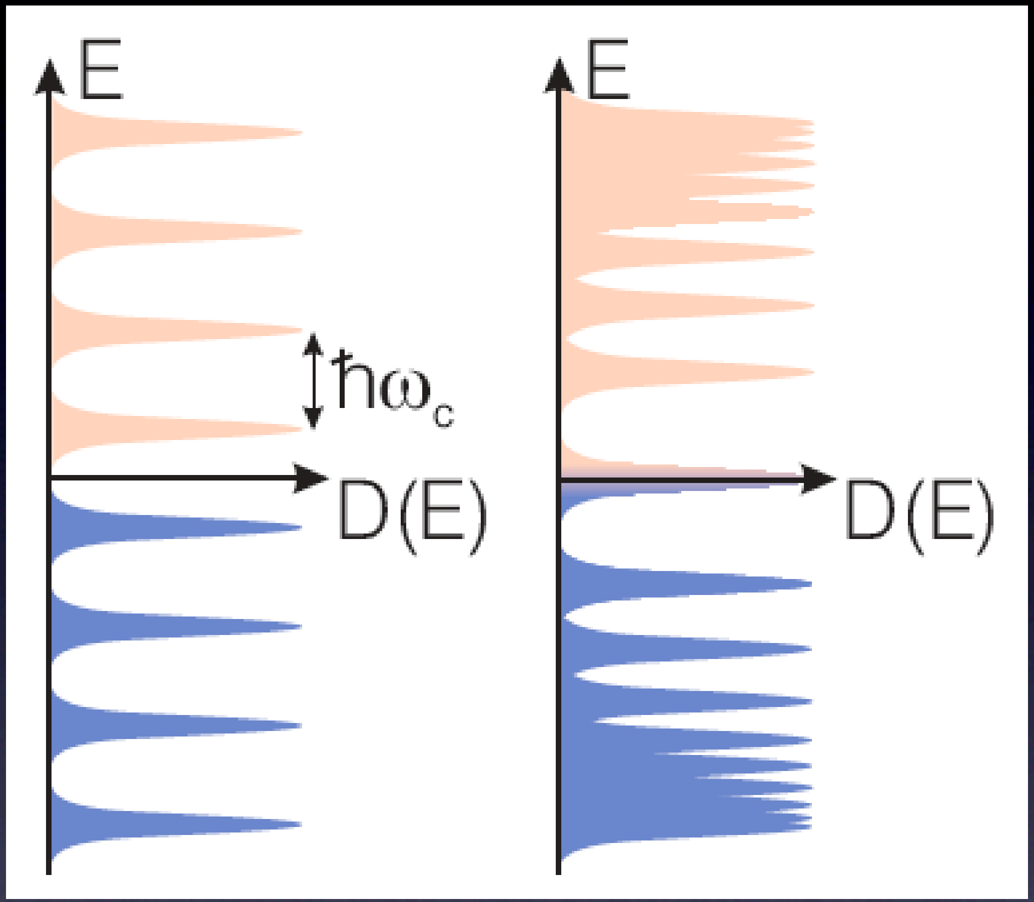



Novoselov et al., Nature 2005

# Anomalous quantum Hall effect

Experimental detection of the massless Dirac spectrum




Landau levels for normal electrons

$$E_n = \left(n + \frac{1}{2}\right) \hbar\omega_c$$

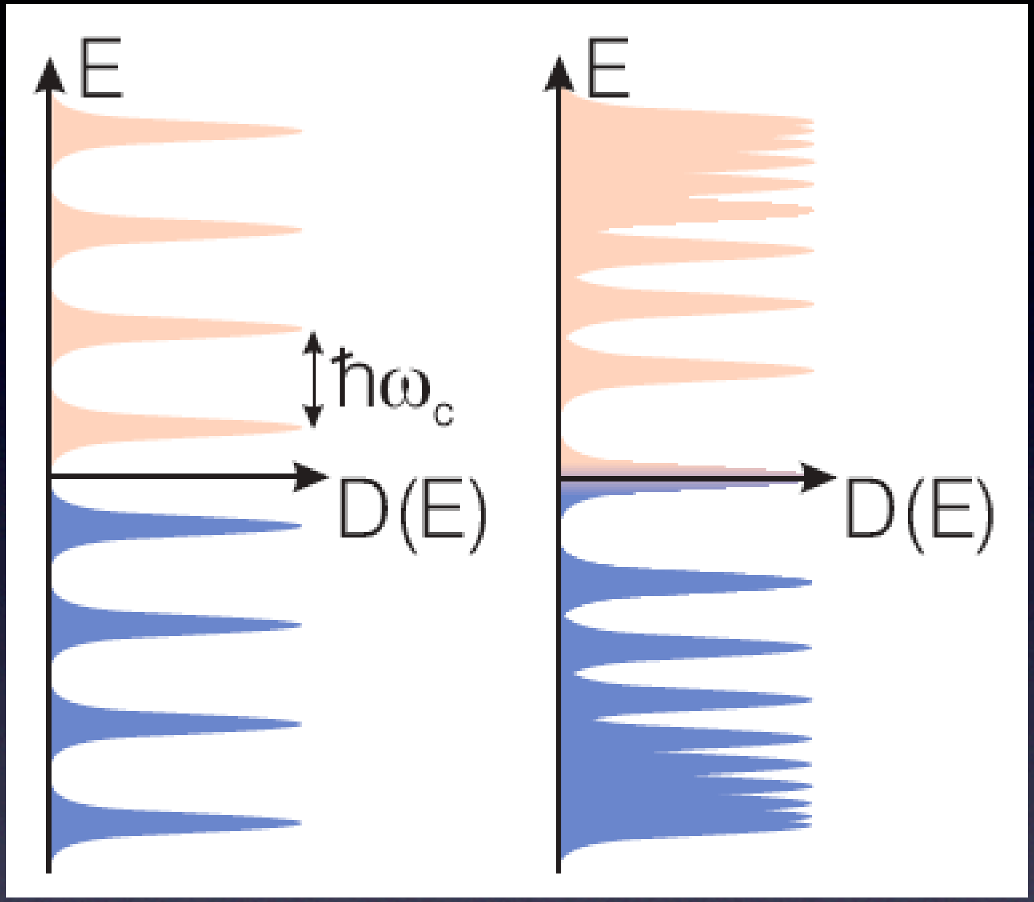
Landau levels for massless Dirac fermions

$$E_n = \pm\sqrt{2e\hbar v^2 |n| B}$$

Quantum Hall effect at room temperature!

# Anomalous quantum Hall effect

Experimental detection
of the massless
Dirac spectrum

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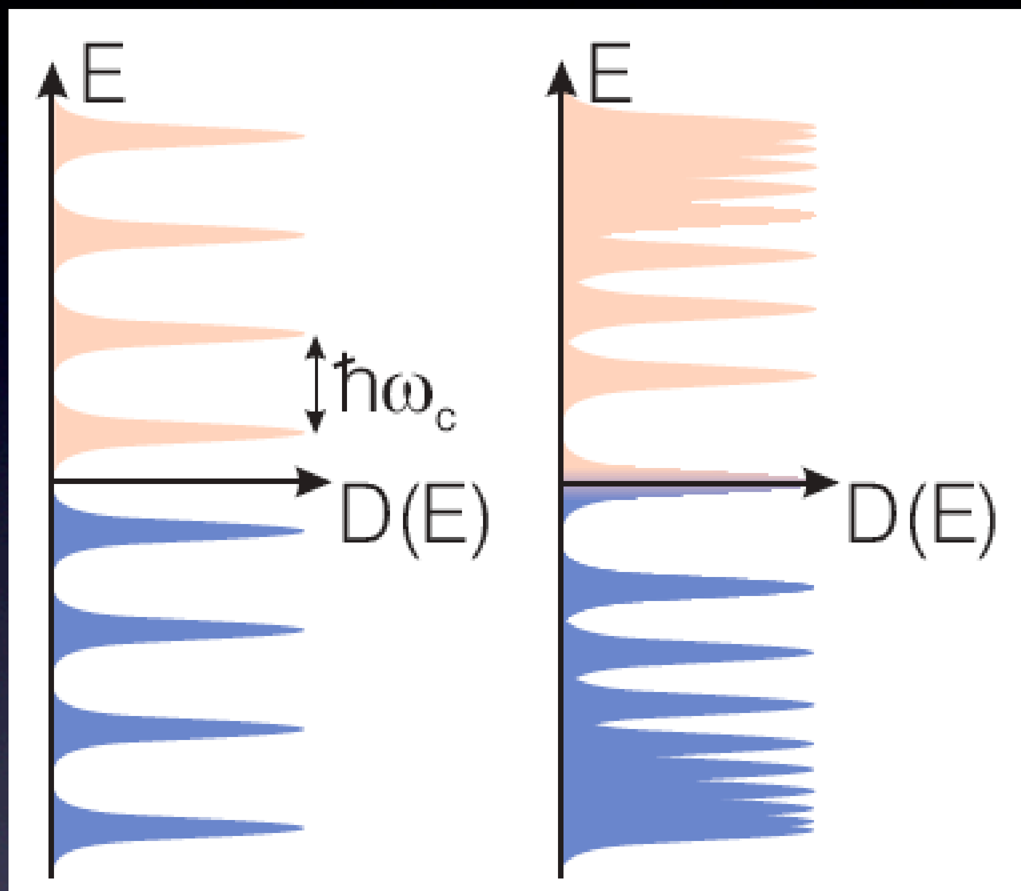


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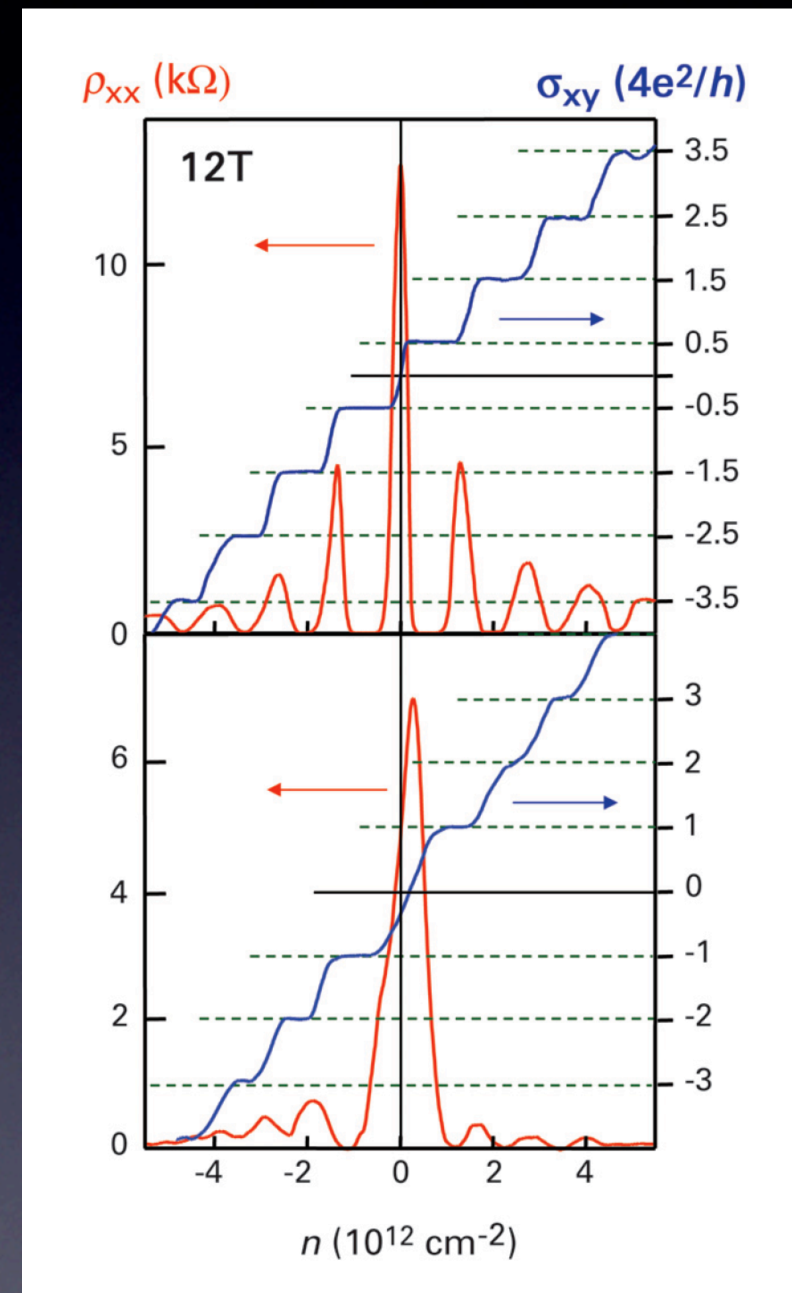



Monolayer graphene

Bilayer graphene

Novoselov et al., Nature 2005
Zhang et al., Nature 2005

# Further aspects

## Massless linear Dirac spectrum

Vanishing DOS at the Dirac points

$$\nu(E) = \frac{|E|}{2\pi\hbar^2 v^2}$$

## Unscreened Interactions

in 2D at the Dirac points

Marginal Fermi liquid

$$\lambda_{\mathrm{scr}} \sim \frac{1}{k_{\mathrm{F}}} \propto \frac{1}{E_{\mathrm{F}}} \to \infty$$

And much more...

PROJEKTTRÄGER FÜR DAS
Bundesministerium
für Bildung
und Forschung
DIP

# Collaborators

WEIZMANN INSTITUTE OF SCIENCE

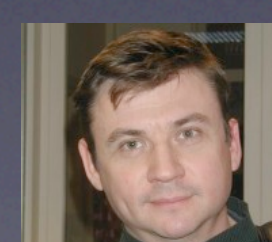
WISSMC-RITA-CT-2003-506095

## Felix von Oppen
FU Berlin

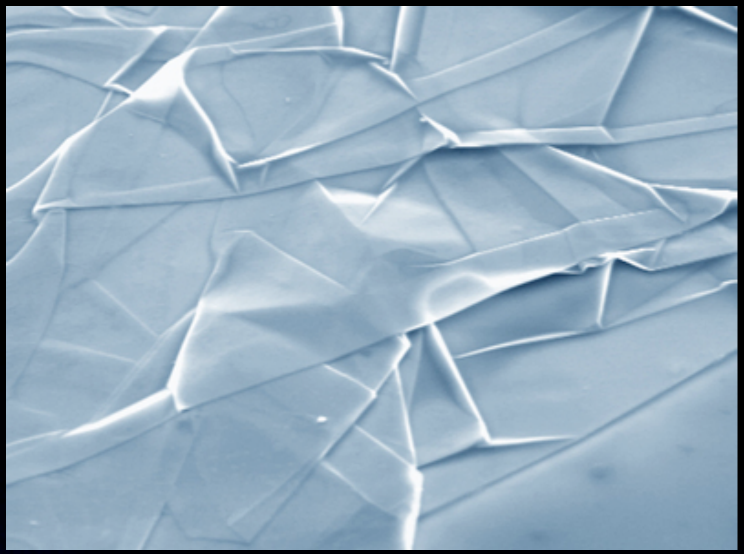
## Leonid Glazman
Yale

## Mikhail Raikh
Utah

## Alex Kamenev
Minnesota



# Conclusions



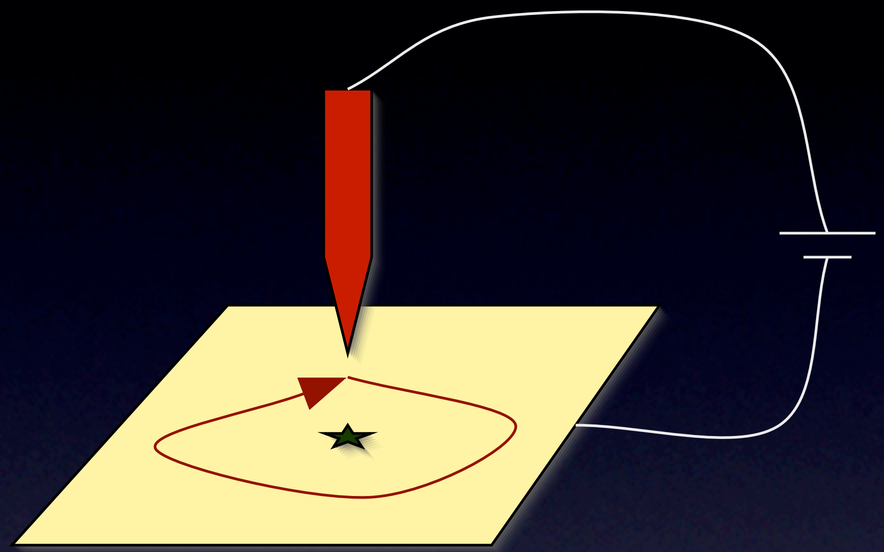
J. Meyer

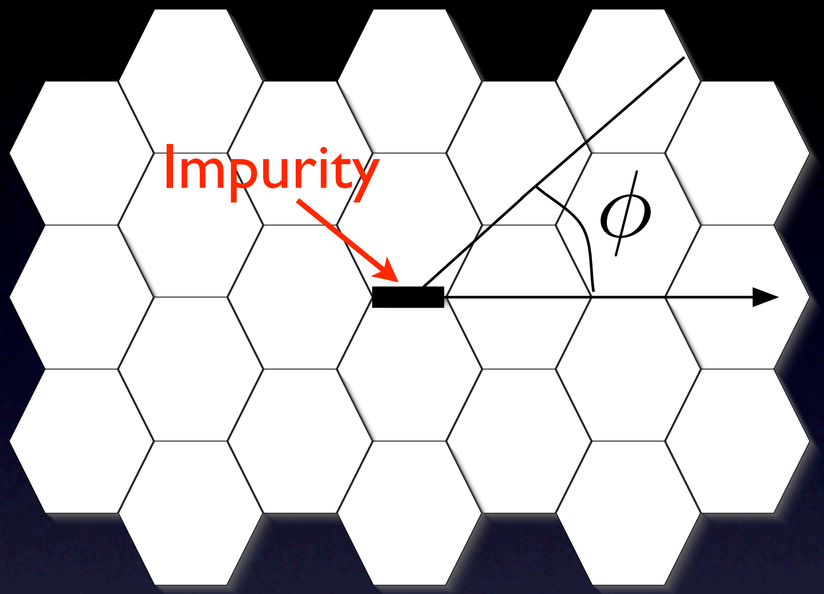
1. Graphene is the first stable 2D crystal
2. First observation of massless Dirac fermions
3. High mobility and finite conductivity with no carriers
4. “New” flexural phonons
5. Scalar potential vs. fictitious gauge fields

E. Mariani, L. Glazman, A. Kamenev and F. von Oppen, PRB 07
E. Mariani and F. von Oppen, PRL 08
E. Mariani, M. Raikh and F. von Oppen, in preparation

# Thank you!

# Tunneling density of states of Graphene






Potential disorder

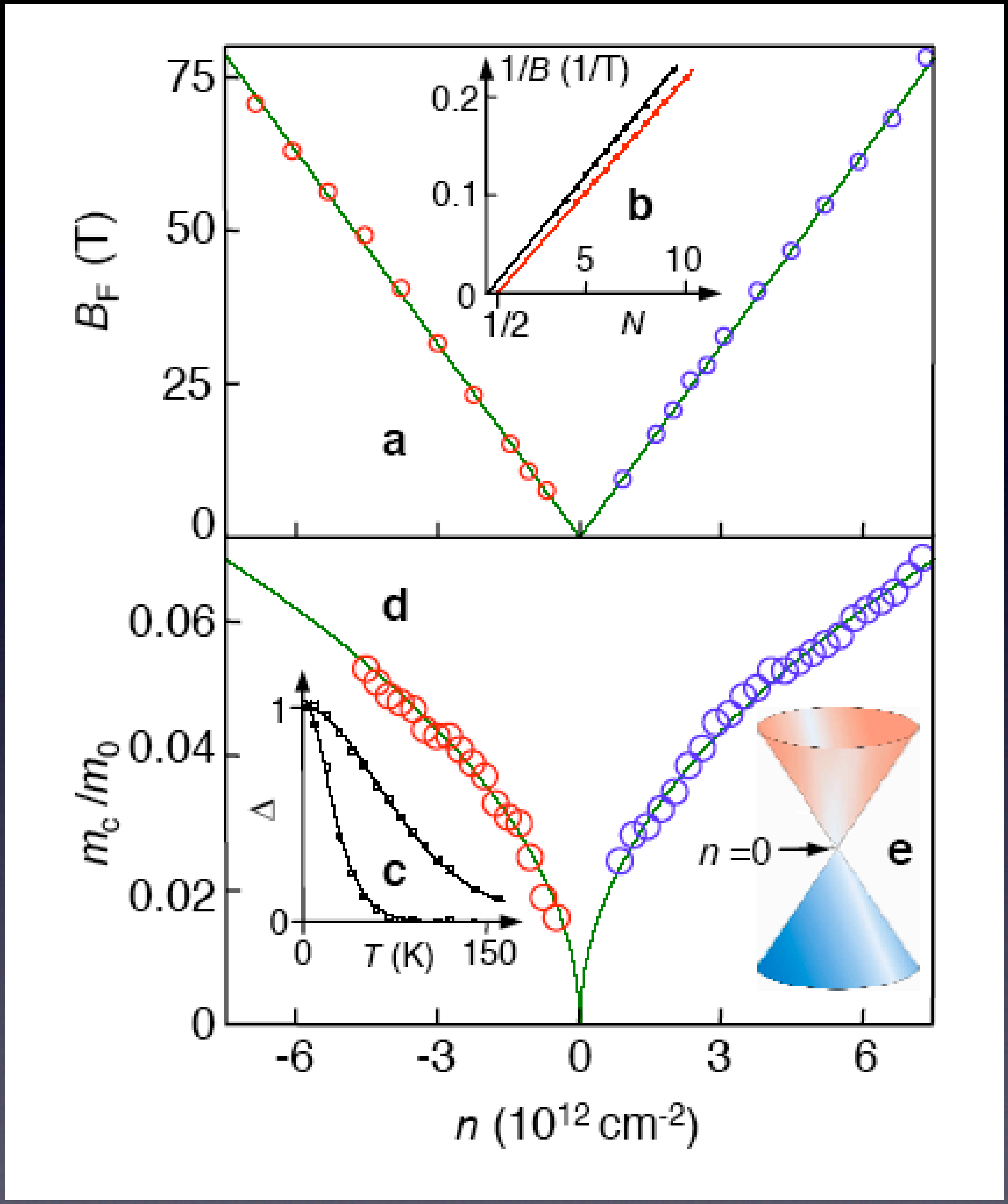
vs

(fictitious) Random gauge field

No angular dependence

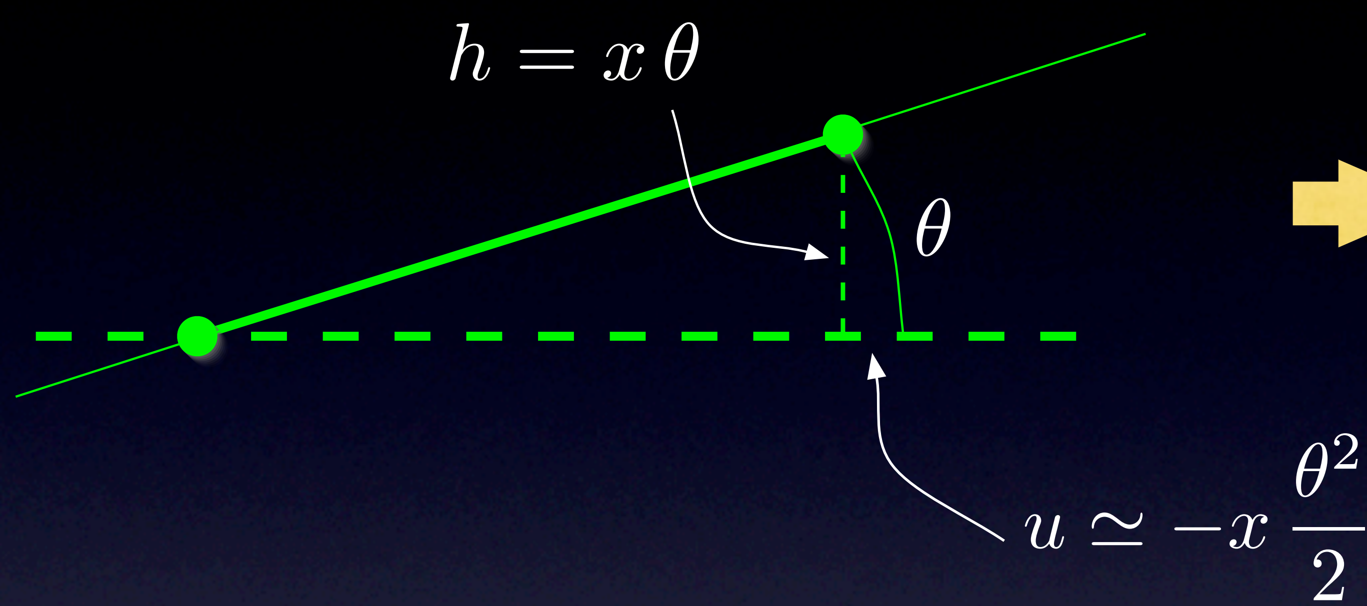
Angular dependence !

# 2D or not 2D?



1/B (1/T)
0.2
0.1
0
1/2
5
10
N
b
a
75
50
25
0
$B_F$ (T)
d
0.06
0.04
0.02
0
$m_c / m_0$
1
Δ
c
0
0
T (K)
150
n =0
e
-6
-3
0
3
6
n ($10^{12}$ cm$^{-2}$)

# Coupling between bending and stretching




Vanishing strain

$$u_{ij} = 0$$

$$F_{\text{stretch}} = \frac{1}{2}\int d\mathbf{r}\,\left[2\mu\,u_{ij}^2 + \lambda\,u_{kk}^2\right]$$

In-plane rigidity

$$F_{\text{bend}} = \frac{1}{2}\int d\mathbf{r}\,\kappa(\nabla^2 h)^2$$

Bending rigidity

$$u_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Strain tensor for a 2D membrane

Maybe write the quartic term as the square of gaussian curvatures

See again the definition of Pab

# upling between bending and stretching

Nelson & Peliti '87

$$F = \frac{1}{2}\int d\mathbf{r}\,\left[\kappa(\nabla^2 h)^2 + 2\mu u_{ij}^2 + \lambda u_{kk}^2\right]$$

$$u_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Integrate out u

$$F_{\rm eff} = \frac{1}{2}\int d\mathbf{r}\left[\kappa\left(\nabla^2 h\right)^2 + \frac{K_0}{4}\left[P^{\perp}_{\alpha\beta}\left(\partial_\alpha h\right)\left(\partial_\beta h\right)\right]^2\right]$$

$$K_0 = 4\mu\frac{\mu+\lambda}{2\mu+\lambda}$$

Maybe write the quartic term as the square of gaussian curvatures

See again the definition of Pab

# upling between bending and stretching

Nelson & Peliti '87

$$F = \frac{1}{2}\int \mathrm{d}\mathbf{r}\,\left[\kappa(\nabla^2 h)^2 + 2\mu u_{ij}^2 + \lambda u_{kk}^2\right]$$

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$$K_0 = 4\mu\frac{\mu+\lambda}{2\mu+\lambda}$$

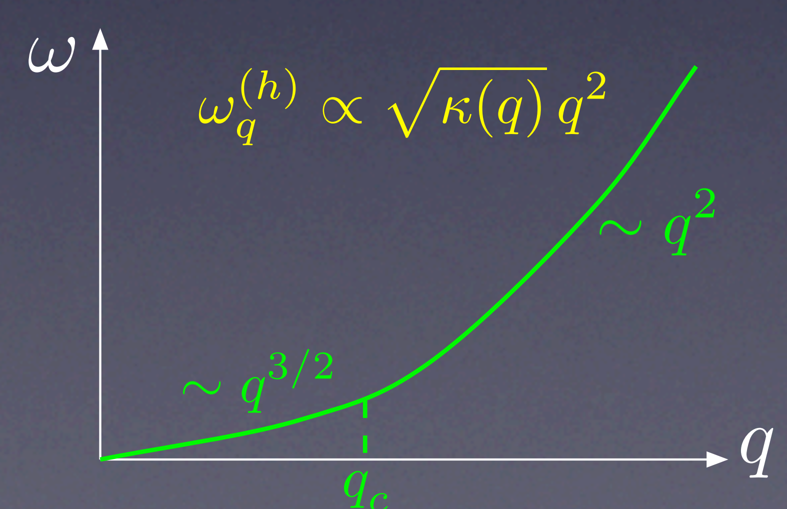
One loop RG

$$\boxed{\kappa(q) = \kappa\sqrt{1 + q_c^2/q^2}}$$

$$q_c = \sqrt{\frac{3K_0 k_{\mathrm{B}} T}{8\pi\kappa^2}}$$

Singularity removed:

Stable flat phase !



# Mott argument

Electrons do not notice any roughness on scales shorter than their wavelength $\lambda$

mean free path is larger than the wavelength

$$l \geq \lambda \quad \Longrightarrow \quad \sigma = \frac{e^2}{\hbar}\frac{l}{\lambda} \sim \frac{e^2}{h}$$

# Mott argument

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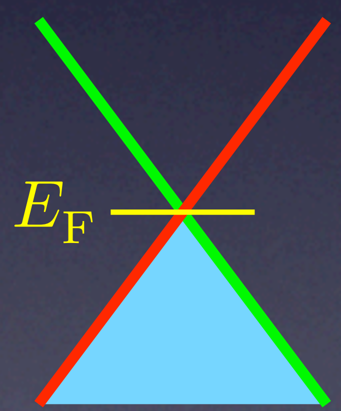
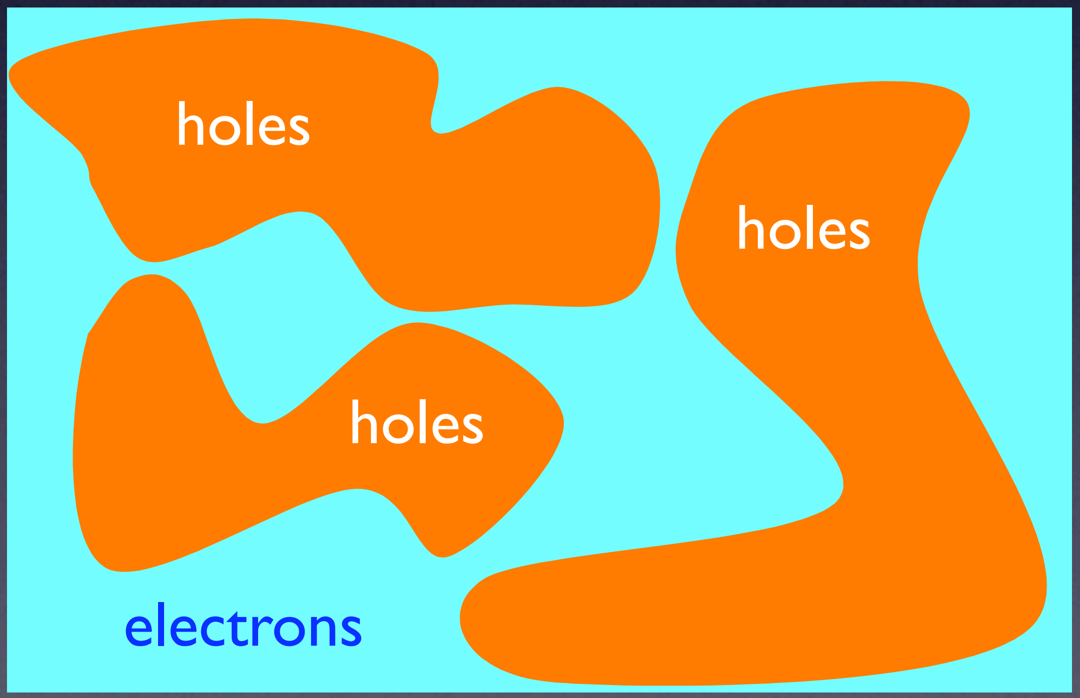
# Puddles in the undoped regime



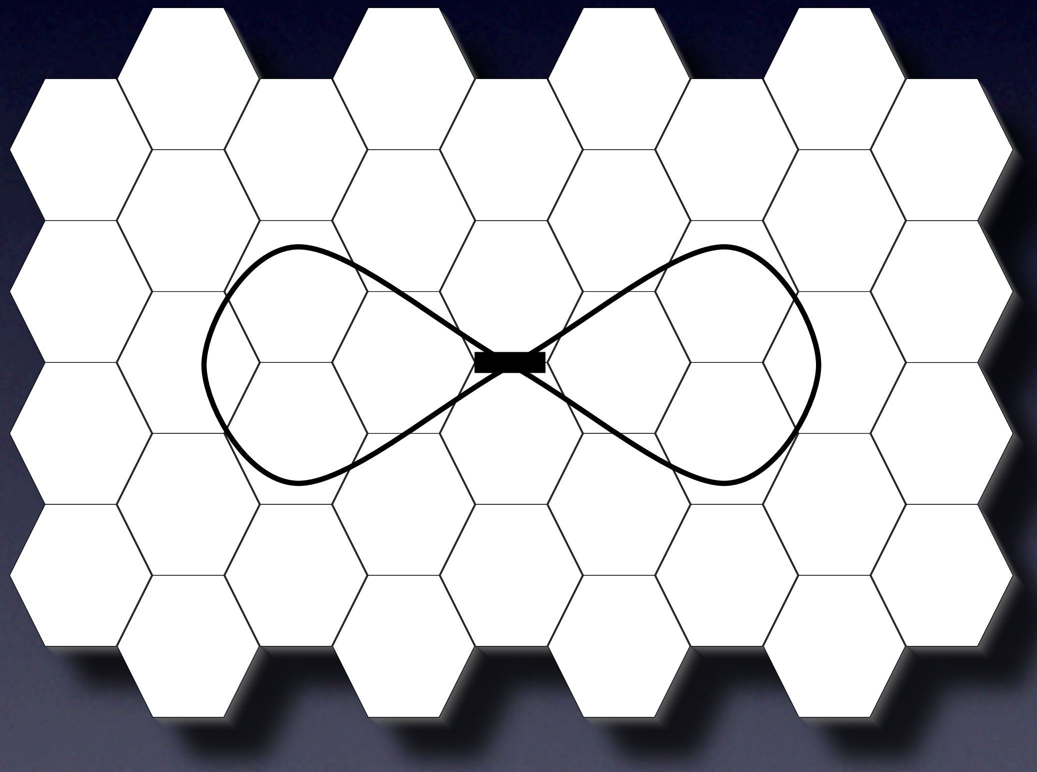

Domain formation due to inhomogeneities or instabilities

Cheianov et al., cond-mat/0706.2968
E. Mariani, M. Raikh and F. von Oppen, in preparation

# Special example

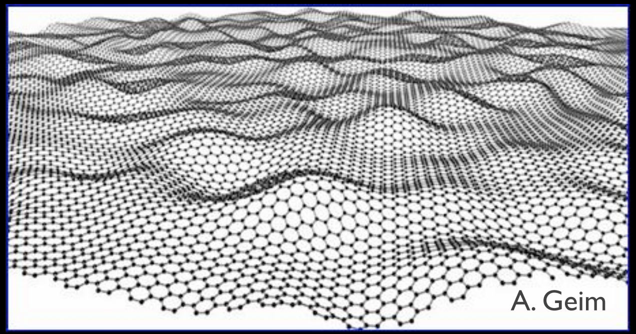
Modified hopping $\delta t$
in one bond produces

$$\frac{\delta\nu_\omega(\mathbf{r})}{\nu_F} \propto (\delta t)^2 \frac{1+\cos^2\phi}{(k_F r)^2}$$

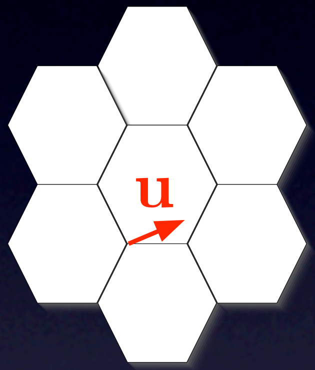


E. Mariani, L. Glazman, A. Kamenev and F. von Oppen, PRB 2007

# Phonons in graphene




## In-plane phonons

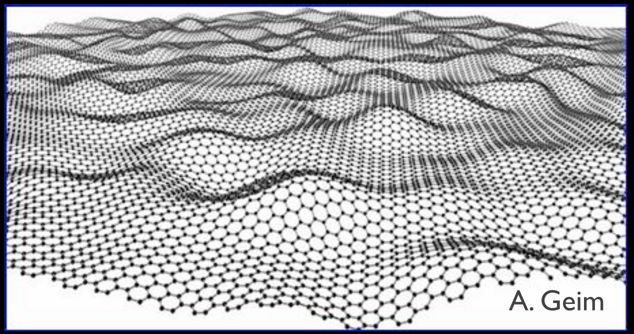



$\mathbf{u}(\mathbf{r})$
In-plane displacement

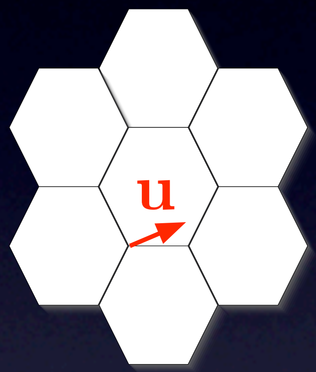
## Flexural phonons


$h(\mathbf{r})$ Out-of-plane displacement

# Phonons in graphene




## In-plane phonons

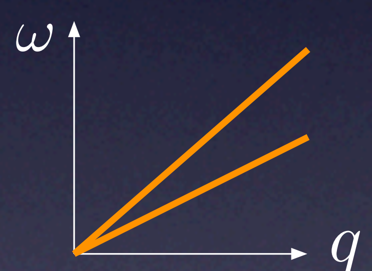



$\mathbf{u}(\mathbf{r})$

In-plane displacement

Linear dispersion

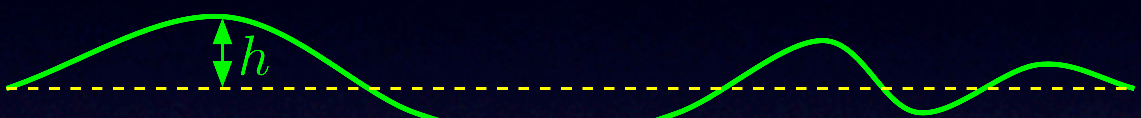
$$\omega^{(l,t)} = v^{(l,t)}\, q$$




Linear coupling

$$\mathbf{A} \propto u_{ij} \sim \nabla u$$

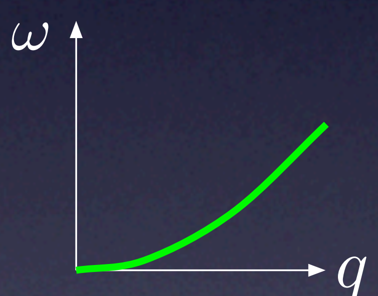

## Flexural phonons




$h(\mathbf{r})$ Out-of-plane displacement

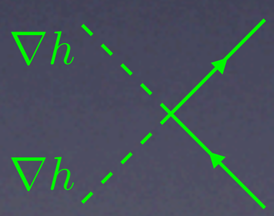
Quadratic dispersion

$$\omega^{(h)} = \alpha\, q^2$$



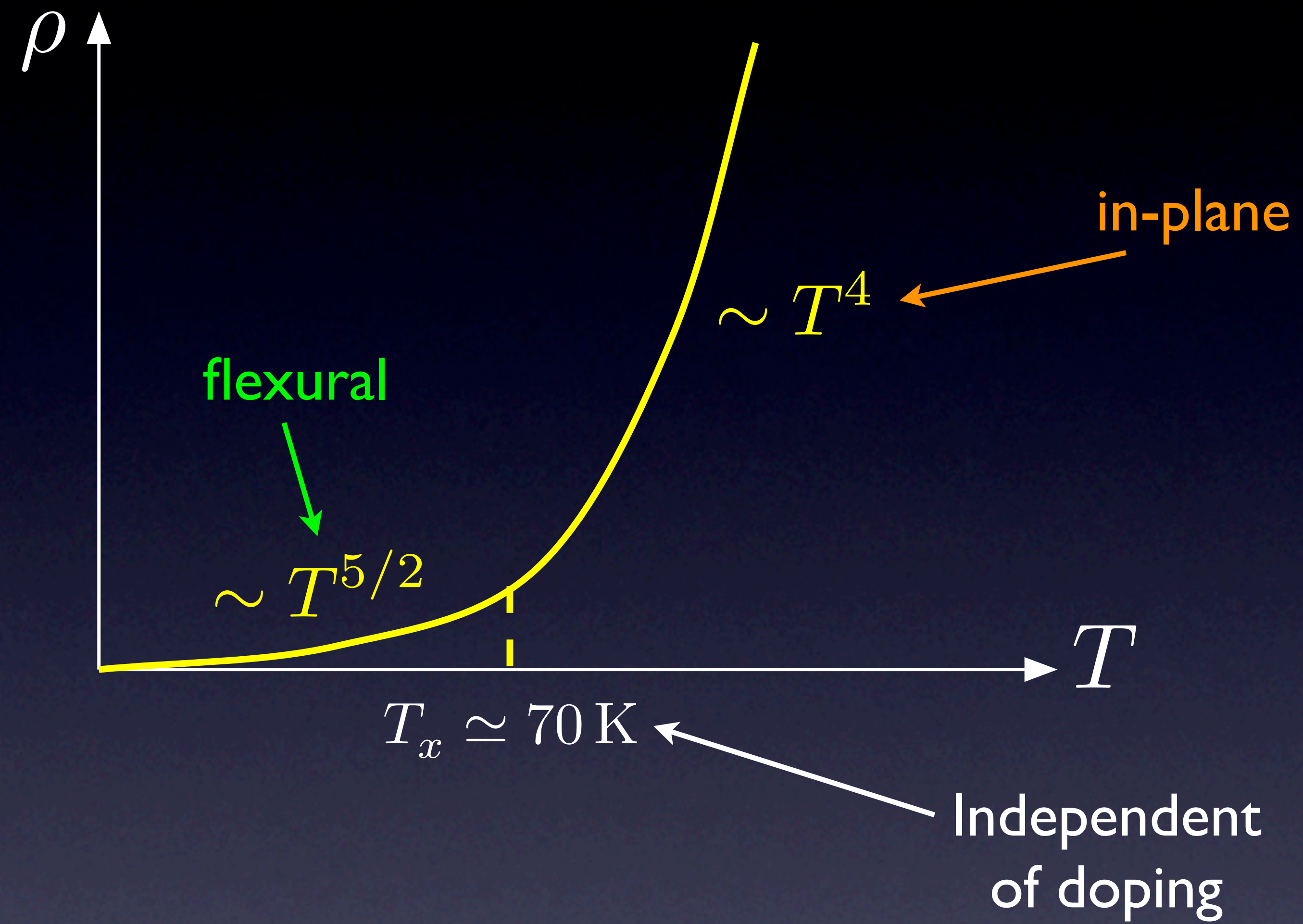

Quadratic coupling

$$\mathbf{A} \propto u_{ij} \sim (\nabla h)^2$$




Who "wins"?

# Temperature-dependent Resistivity




At low temperatures ( $T < 70\,\mathrm{K}$ ) the phonon contribution to the resistivity is dominated by flexural modes and scales as $T^{5/2} \ln T$

E. Mariani and F. von Oppen, PRL 08