Environmental noise reduction for holonomic quantum gates

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Classical Computation



Classical Computation

Quantum Computation (QC)



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Quantum Computation (QC)

Power of QC: Ex. Quantum Parallelism

Logical Strings: 00001, 01010, 11010 etc.

Classical Computation

Quantum Computation (QC)

Power of QC: Ex. Quantum Parallelism

Superposition of "Logical Strings":

$$|\psi\rangle = |00001\rangle + |01010\rangle + |11010\rangle$$

Classical Computation

Quantum Computation (QC)

Power of QC: Ex. Quantum Parallelism

Stability against control and environmental noise?

Systems and Approaches to QC

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Almost any quantum system can be used to perform QC

- NMR, trapped ions, atom in cavity, Josephson Junction, Quantum Dots
- Quantum Dots with laser control

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Many way to perform QC

- Construction of "dynamical gates"
- Adiabatic Quantum Computation
- Geometric Quantum Computation

Time dependent Hamiltonian $H(0) = H(T_{fin})$

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- slow evolution

at the end of the evolution

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 φ is the geometric (Berry) phase

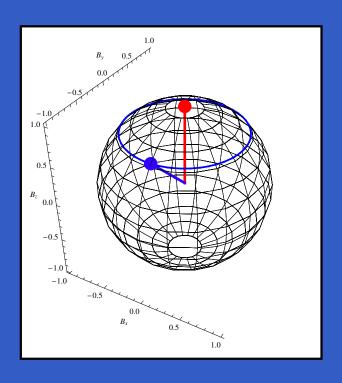
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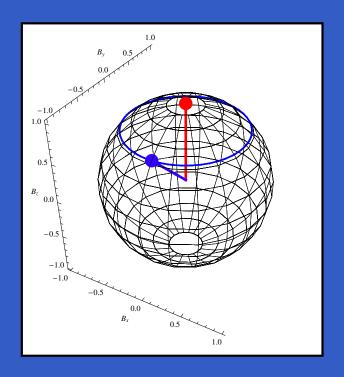
$$|\psi(T_{fin})\rangle = e^{i\varphi}|\psi(0)\rangle$$
 so the geometric (Berry) phase

Holonomic Quantum Computation: geometric approach with generalized Berry phase

Berry Phase



Berry Phase



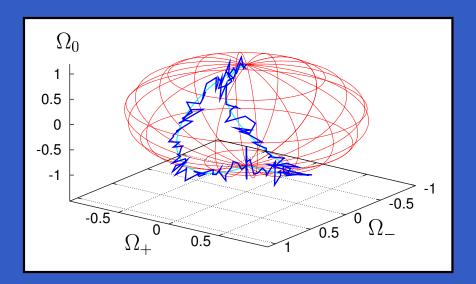
Evolution is driven by H(t): a curve in the parameter space is associated to a dynamical evolution

Noise in QC

Geometric Quantum Computation is robust against controll errors

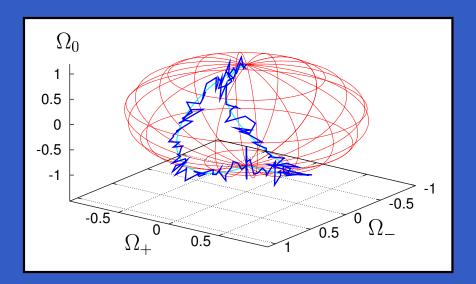
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Control errors cancel out

System interacting with a bath of bosons

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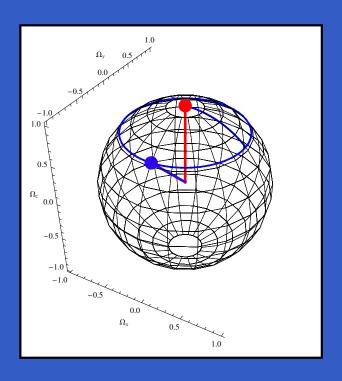
Analytical evaluation of the error induced by the environment

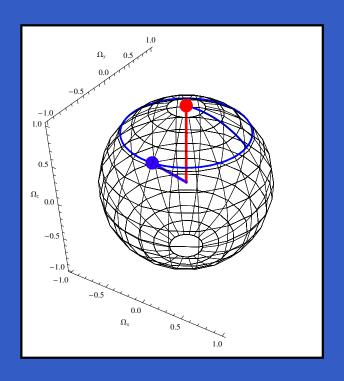
HQC operator depends on the solid angle spanned in the parameter space

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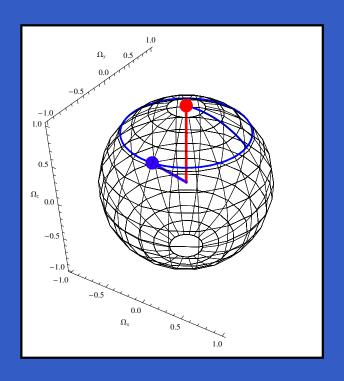
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- Effect of the environment depends on the dynamic of the system
- ⇒ we can separate the operator applied from the dynamical evolution
- find a weakly affected evolution producing the same geometric operator





- the North Pole is a noise free point ⇒ Best loops are those near the North Pole
- Spherical Cap covered several time



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Once fixed

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Once fixed

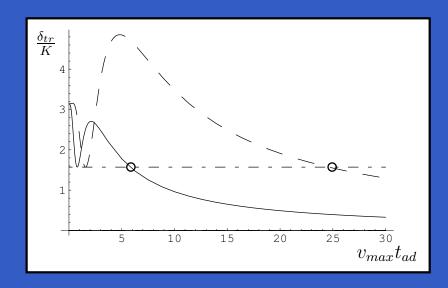
- speed in the parameter space
- the solid angle (logic operator)
- long evolutions allows to have a small spherical cap

Once fixed

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- the solid angle (logic operator)
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the noise effect decrease decrease with time evolution $\delta \propto \frac{1}{4}$



Geometrical Quantum Computation

Robust against controll error

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- Separation of Geometric Operator from Dynamical (Noise) Effect

Geometrical Quantum Computation

- Robust against controll error
- Possibility to reduce the environmental noise
- Separation of Geometric Operator from Dynamical (Noise) Effect
- Choice of the best loop to produce robust gates