

HYPERKÄHLER QUOTIENT & LUMPS IN $SO(N)$ AND $USp(N)$ GAUGE THEORIES

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Motivations

- ▶ The low energy effective theory on the Higgs branch in $\mathcal{N} = 2$ sQCD is hyperKähler
- ▶ ... $\mathcal{N} = 1$ sQCD is Kähler
- ▶ Topology, metric
 - low energy effective action
- ▶ Topology
 - topological excitations in the gauge theories
 - non-perturbative effects
- ▶ In extension of the line of research of Nitta et.al. (BPS domain walls) and Eto et.al. (BPS domain walls and SO/USp vortices)

Supersymmetric non-linear σ -model

We want to write down the low energy effective theory of a supersymmetric model.

Taking the Kähler potential describing the theory in question in terms of chiral superfields :

$$K = K(\Phi, \bar{\Phi}) \quad (1)$$

the non-linear σ -model can be written as

$$\mathcal{L} = -g_{i\bar{i}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{i}} - i g_{i\bar{i}} \bar{\psi}^{\bar{i}} \bar{\sigma}^\mu \mathcal{D}_\mu \psi^i + \frac{1}{4} R_{ijkl} \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}} \quad (2)$$

with

$$g_{i\bar{i}} \equiv \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \bar{\phi}^{\bar{i}}} K \quad (3)$$

being the metric on the Kähler manifold and

$\mathcal{D}_\mu \psi^i \equiv \partial_\mu \psi^i + \Gamma_{jk}^i \partial_\mu \phi^k \psi^j$ with corresponding connection Γ and Riemann tensor R

Kähler quotient for a gauge theory

Consider an $\mathcal{N} = 1$ gauge theory with a **simple** gauge group G with N_F chiral superfields in the fundamental representation \square of the gauge group along with a vector multiplet V'

$$K = \text{Tr} \left[Q Q^\dagger e^{-V'} \right] \quad (4)$$

notice that the invariance of the Lagrangian is $G^{\mathbb{C}}$, e.g. for $SU(N)$ it is $SU(N)^{\mathbb{C}} = SL(N, \mathbb{C})$ or for $U(N)$ it is $U(N)^{\mathbb{C}} = GL(N, \mathbb{C})$

$$Q \rightarrow e^{i\Lambda} Q \quad (5)$$

$$e^{V'} \rightarrow e^{i\Lambda} e^{V'} e^{-i\Lambda^\dagger}, \quad \text{with } e^{i\Lambda} \in G^{\mathbb{C}} \quad (6)$$

we would now like to solve the Kähler quotient i.e. integrating out the vector multiplet

Roads to a low energy effective theory

- ▶ 1st road
 - ▶ fix the gauge, i.e. Wezz-Zumino gauge \leftrightarrow resolve the D-term constraints
 - ▶ take the limit $g \rightarrow \infty$
 - ▶ mod out the gauge symmetry
- ▶ 2nd road
 - ▶ take the limit $g \rightarrow \infty$
 - ▶ mod out the **full complexified** gauge symmetry
- ▶ 3rd road
 - ▶ write down the **holomorphic invariants**
 - ▶ identify algebraic relations (e.g. the Plücker relation)

A comment in store is that the validity is strongly dependent on being in the full Higgs phase, i.e. full rank of Q

A quick example

► $SU(N)$

$$K = N \left[\det QQ^\dagger \right]^{\frac{1}{N}} \quad 2^{\text{nd}} \text{ road} \quad (7)$$

$$= N \left[\sum_{\langle A \rangle} |B^{\langle A \rangle}|^2 \right]^{\frac{1}{N}} \quad 3^{\text{rd}} \text{ road} \quad (8)$$

$\det QQ^\dagger = 0$ resembles the Coulomb branch

► $U(N)$: the singularity can be lifted by an FI parameter ξ

$$K = \text{Tr} \left[QQ^\dagger e^{-V'} e^{-V_e} \right] + \xi V_e \quad (9)$$

$$= \frac{\xi}{N} \log \det QQ^\dagger = \frac{\xi}{N} \log \left[\sum_{\langle A \rangle} |B^{\langle A \rangle}|^2 \right] \quad (10)$$

with V_e being a $U(1)$ vector multiplet

The Kähler potential can be written as

$$K = \text{Tr} \left[QQ^\dagger e^{-V'} \right] \quad (11)$$

where the following constraints on the gauge fields kick in

$$V'^T J + J V' = 0 \quad \Leftrightarrow \quad e^{-V'^T} J e^{-V'} = J \quad (12)$$

with $J = \begin{pmatrix} \mathbf{0} & \mathbf{1}_M \\ \epsilon \mathbf{1}_M & \mathbf{0} \end{pmatrix}$ where $\epsilon = \pm 1$ for SO/USp gauge group

- ▶ The D-flatness condition turns out to be quite difficult
- ▶ so we jump to the 2nd road

A trick : enlarging the algebra

Let's take the vector multiplet to be in $SU(N) \ni e^{-V'}$

$$K = \text{Tr} \left[QQ^\dagger e^{-V'} + \lambda \left(e^{-V'^T} J e^{-V'} - J \right) \right] \quad (13)$$

- ▶ e.o.m.s for λ solve the constraint of the algebra of SO/USp
- ▶ e.o.m.s for V' include λ , but λ can be eliminated from these

The solution is obtained as

$$X = \sqrt{QQ^\dagger} e^{-V'} \sqrt{QQ^\dagger} \quad (14)$$

$$X^2 = \left(Q^T J \sqrt{QQ^\dagger} \right)^\dagger \left(Q^T J \sqrt{QQ^\dagger} \right) \quad (15)$$

$$K = \text{Tr} X \quad 2^{\text{nd}} \text{ road} \quad (16)$$

The solution in terms of holomorphic invariants

Using another trick, i.e. rewriting a color trace as a flavor trace

$$\mathrm{Tr}_C \sqrt{AA^\dagger} = \mathrm{Tr}_F \sqrt{A^\dagger A} \quad (17)$$

we can write the solution as

$$K = \mathrm{Tr}_F \sqrt{MM^\dagger} \quad 3^{\mathrm{rd}} \text{ road} \quad (18)$$

with $M = Q^T J Q$ being the **meson field**

Target spaces

- ▶ USp

$$\mathcal{M}_{USp} = \{M \mid M \in \mathbb{C}^{N_F} \times \mathbb{C}^{N_F}, M^T = -M, \quad (19)$$

$$\text{rank} M = 2M_C\} \quad (20)$$

- ▶ reflects the fact that there are no independent baryons in USp gauge theories

- ▶ SO

$$\mathcal{M}_{SO} = \{M, B^{\langle A \rangle} \mid M \in \mathbb{C}^{N_F} \times \mathbb{C}^{N_F}, M^T = M, \quad (21)$$

$$\det M^{\langle A \rangle \langle B \rangle} = (\det J) B^{\langle A \rangle} B^{\langle B \rangle}, \quad (22)$$

$$N_C - 1 \leq \text{rank} M \leq N_C\} \quad (23)$$

Extending the gauge group with an overall $U(1)$

Using the result from previously together with the e.o.m. for V_e , the Kähler potential can now be written as

$$K = \text{Tr} \left[Q Q^\dagger e^{-V'} e^{-V_e} \right] + \xi V_e \quad (24)$$

$$= \frac{\xi}{N} \log \text{Tr}_F \sqrt{M M^\dagger} \quad (25)$$

with V_e being a $U(1)$ vector multiplet

$\mathcal{N} = 2$ susy : hyperKähler quotient for $SO(N)$ and $USp(2M = N)$

Considering an $\mathcal{N} = 2$ gauge theory, we can write the Kähler potential as

$$\tilde{K} = \text{Tr} \left[QQ^\dagger e^{-V'} + \tilde{Q}^\dagger \tilde{Q} e^{V'} + \lambda \left(e^{-V'^T} J e^{-V'} - J \right) \right] \quad (26)$$

with the superpotential

$$\mathcal{W} = \text{Tr} \left[Q \tilde{Q} \Sigma' + \chi \left(\Sigma'^T J + J \Sigma' \right) \right] \quad (27)$$

Using the algebra of SO/USp , $e^{V'^T} = J^T e^{-V'} J$ and the Kähler potential can be written as

$$\tilde{K} = \text{Tr} \left[QQ^\dagger e^{-V'} + J^T e^{-V'} J (\tilde{Q}^\dagger \tilde{Q})^T + \lambda \left(e^{-V'^T} J e^{-V'} - J \right) \right] \quad (28)$$

$\mathcal{N} = 2$ susy : hyperKähler quotient for $SO(N)$ and $USp(2M = N)$ page II

Defining now a field with $2N_F$ flavors

$$Q = (Q, J\tilde{Q}^T) \quad (29)$$

the Kähler potential can be written as

$$\tilde{K} = \text{Tr} \left[QQ^\dagger e^{-V'} + \lambda \left(e^{-V'^T} J e^{-V'} - J \right) \right] \quad (30)$$

and the $\mathcal{N} = 1$ solution readily applies!

$$\tilde{K} = \text{Tr}_F \sqrt{\mathcal{M}\mathcal{M}^\dagger} \quad (31)$$

where the meson field now is

$$\mathcal{M} = Q^T J Q \quad (32)$$

Constraint coming from the superpotential

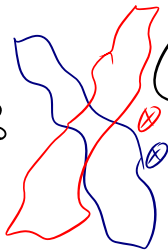
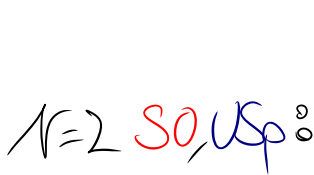
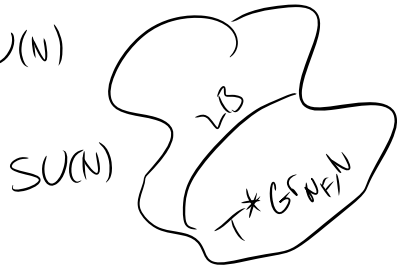
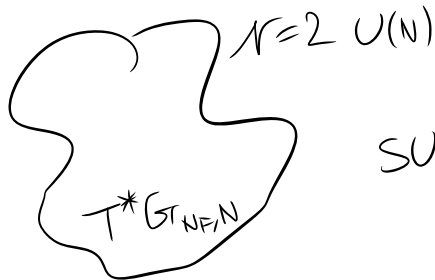
The following constraint

$$Q\tilde{J}Q^T = 0 \quad (33)$$

where $\tilde{J} = \begin{pmatrix} \mathbf{0} & \mathbf{1}_M \\ -\epsilon\mathbf{1}_M & \mathbf{0} \end{pmatrix}$ shows the well-known result, that

- ▶ An $SO(N_C)$ gauge theory has a $USp(2N_F)$ flavor symmetry
- ▶ A $USp(2M_C)$ gauge theory has an $SO(2N_F)$ flavor symmetry

Resultant manifolds



Conclusion

- ▶ Explicit construction of the up-to-now unknown Kähler and hyperKähler metrics for $SO/U\text{Sp}$ gauge groups.