

Hadron-Hadron (Dipole-Dipole) High Energy Scattering from Lattice QCD

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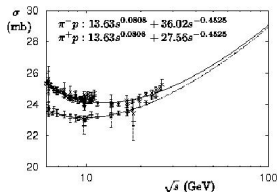
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Convegno Informale di Fisica Teorica - Sestri 2008
June 06, 2008

- 1 High Energy Hadron-Hadron Scattering
 - Nonperturbative Approach To High Energy Soft Scattering
 - Analytic Continuation Into Euclidean Space
- 2 Wilson Loop Correlators On the Lattice
 - Overview
 - Numerical Results

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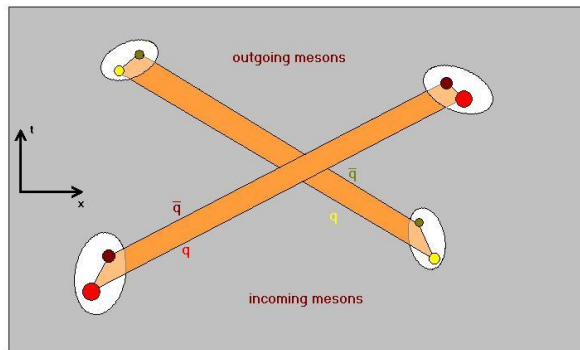
High Energy Scattering and QCD



$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=1}^6 \bar{\Psi}^{(f)} (i\not{D} - m_f) \Psi^{(f)}$$

- hadron-hadron total cross sections rise with energy
 - ▶ Pomeron model
 $\Rightarrow \sigma_{tot}(s) \sim s^{0.08}$
 - ▶ Froissart bound
 $\Rightarrow \sigma_{tot}(s) \leq \frac{\pi}{m_\pi^2} \log^2\left(\frac{s}{s_0}\right)$
- QCD is the fundamental theory of strong interactions \Rightarrow first principles study
- soft high energy scattering: $s \rightarrow \infty$, $|t| \leq (1\text{GeV})^2$
 - ▶ perturbation theory not reliable
 - ▶ non perturbative approach
(Nachtmann, 1991)

Eikonal Approximation



- partonic description of hadrons
- partons as in/out states
- lightlike trajectories approximately unchanged in the process

Meson-Meson (Dipole-Dipole) Scattering

- meson-meson elastic scattering amplitude in the eikonal approximation (Dosch *et al.*, 1994)

$$\begin{aligned} \mathcal{M}_{(12 \rightarrow 12)}(\mathbf{s}, t) &= -i 2s \\ &\times \int d^2 \vec{z}_\perp e^{i \vec{q}_\perp \cdot \vec{z}_\perp} \int d^2 \vec{R}_{1\perp} |\psi_1(\vec{R}_{1\perp})|^2 \int d^2 \vec{R}_{2\perp} |\psi_2(\vec{R}_{2\perp})|^2 \\ &\times \mathcal{G}_M(\chi \rightarrow +\infty; T \rightarrow +\infty; \vec{z}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}) \quad [t = -|\vec{q}_\perp|^2] \end{aligned}$$

- Wilson loop correlation functions

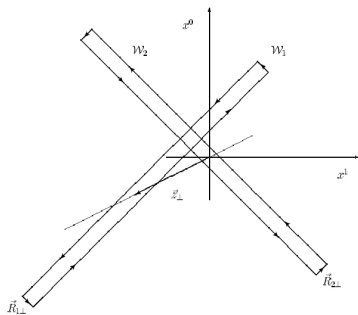
$$\mathcal{G}_M(\chi; T; \vec{z}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}) = \frac{\langle \mathcal{W}[C_1] \mathcal{W}[C_2] \rangle}{\langle \mathcal{W}[C_1] \rangle \langle \mathcal{W}[C_2] \rangle} - 1$$

$$\mathcal{W}[C] \equiv \frac{1}{N_c} \text{tr} \mathcal{P} \exp \left\{ -ig \oint_C A_\mu(X) dX^\mu \right\}$$

$$\langle \mathcal{O}[A] \rangle \equiv Z^{-1} \int [dA] \mathcal{O}[A] \det \mathcal{F}[A] e^{iS_g[A]}$$

- gauge invariant
- finite $T \rightarrow \infty$ limit expected
- $C_M = \lim_{T \rightarrow \infty} \mathcal{G}_M$

Minkowskian Wilson Loops



$$p_1 \cdot p_2 = m^2 \cosh \chi$$
$$\chi \underset{s \rightarrow \infty}{\simeq} \log \frac{s}{m^2}$$

- longitudinal sides $\Rightarrow q - \bar{q}$ trajectories ($\tau \in [-T, T]$)

$$C_1 \rightarrow X_{(\pm 1)}(\tau) = z + \frac{p_1}{m} \tau \pm \frac{R_1}{2}$$

$$C_2 \rightarrow X_{(\pm 2)}(\tau) = \frac{p_2}{m} \tau \pm \frac{R_2}{2}$$

$$\frac{p_{1,2}}{m} = \left(\cosh \frac{\chi}{2}, \pm \sinh \frac{\chi}{2}, \vec{0}_\perp \right)$$

- closed at $\tau = \pm T$ by straight “links” in the transverse plane

$$R_i = (0, 0, \vec{R}_{i\perp}) \quad z = (0, 0, \vec{z}_\perp)$$

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Euclidean Correlation Functions

- non perturbative techniques available in Euclidean space \Rightarrow Euclidean correlation functions

$$\mathcal{G}_E(\theta; T; \vec{z}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}) = \frac{\langle \widetilde{\mathcal{W}}[\tilde{\mathcal{C}}_1] \widetilde{\mathcal{W}}[\tilde{\mathcal{C}}_2] \rangle_E}{\langle \widetilde{\mathcal{W}}[\tilde{\mathcal{C}}_1] \rangle_E \langle \widetilde{\mathcal{W}}[\tilde{\mathcal{C}}_2] \rangle_E} - 1$$

$$\widetilde{\mathcal{W}}[\tilde{\mathcal{C}}] \equiv \frac{1}{N_c} \text{tr} \mathcal{P} \exp \left\{ -ig \oint_{\tilde{\mathcal{C}}} A_{E\mu}(X_E) dX_{E\mu} \right\}$$

- paths in Euclidean space

$$\begin{aligned} \tilde{\mathcal{C}}_1 \rightarrow X_{E(\pm 1)}(\tau) &= z_E + \frac{p_{1E}}{m} \tau \pm \frac{R_{1E}}{2} & \frac{p_{1,2E}}{m} &= \left(\cos \frac{\theta}{2}, \vec{0}_\perp, \pm \sin \frac{\theta}{2} \right) \\ \tilde{\mathcal{C}}_2 \rightarrow X_{E(\pm 2)}(\tau) &= \frac{p_{2E}}{m} \tau \pm \frac{R_{2E}}{2} & R_{iE} &= (0, \vec{R}_{i\perp}, 0) \\ & & z_E &= (0, \vec{z}_\perp, 0) \end{aligned}$$

▶ Euclidean functional integral

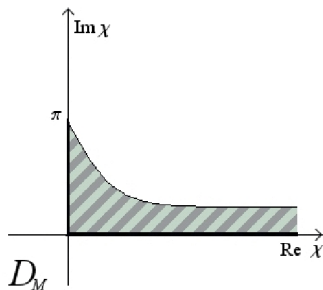
▶ finite $T \rightarrow \infty$ limit expected
 $\mathcal{C}_E = \lim_{T \rightarrow \infty} \mathcal{G}_E$

Analytic Continuation Relations

- correlation functions in Minkowski space can be reconstructed from Euclidean correlation functions (Meggiolaro, 2005)

$$\mathcal{G}_M(\chi; T) = \mathcal{G}_E(\theta \rightarrow -i\chi; T \rightarrow iT)$$

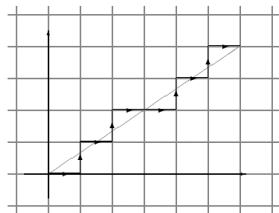
$$\mathcal{C}_M(\chi) = \mathcal{C}_E(\theta \rightarrow -i\chi)$$



- non perturbative calculations:
 - instantons (Shuryak, Zahed, 2001)
 - AdS/CFT (Janik, Peschanski, 2000)
 - SVM (Shoshi *et al.*, 2003)
- numerical approach via lattice gauge theory

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Loop Construction



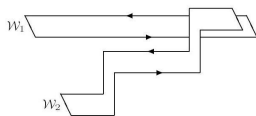
- Bresenham prescription: evaluate the loop sides on the lattice path that minimizes the distance from the continuum path
- $\widetilde{\mathcal{W}}_L(\vec{l}_{\parallel}, \vec{r}_{\perp}; n)$: center in n , sides l (longitudinal plane) and r (transverse plane)

- lattice Wilson loop correlators ($d = (0, \vec{d}_{\perp}, 0)$)

$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) = \frac{\langle \widetilde{\mathcal{W}}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d) \widetilde{\mathcal{W}}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0) \rangle_E}{\langle \widetilde{\mathcal{W}}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d) \rangle_E \langle \widetilde{\mathcal{W}}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0) \rangle_E} - 1$$

- ▶ (full) rotation invariance is broken \Rightarrow explicit dependence on the lattice vectors $l_i, r_i, i = 1, 2$ and d

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- ▶ (full) rotation invariance is broken \Rightarrow explicit dependence on the lattice vectors $l_i, r_i, i = 1, 2$ and d

- in the continuum limit rotation invariance is restored ($L_i = \sqrt{\vec{l}_{i\parallel}^2}$)

$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp})$$
$$\underset{a \rightarrow 0}{\simeq} \mathcal{G}_E(\theta; aL_1/2, aL_2/2; a\vec{d}_{\perp}, a\vec{r}_{1\perp}, a\vec{r}_{2\perp}) + \mathcal{O}(a)$$

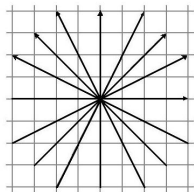
- ▶ to keep as low as possible the $\mathcal{O}(a)$ correction \Rightarrow keep one of the two loops on-axis and rotate only the other one
- ▶ lattice spacing a related to the bare coupling constant g through the renormalisation group equation ($N_c = 3, N_f = 0$)

$$a(\beta) = \frac{1}{\Lambda_L} \left(\frac{8}{33} \pi^2 \beta \right)^{\frac{51}{121}} \exp \left[-\frac{4}{33} \pi^2 \beta \right], \quad \beta = \frac{6}{g^2}$$

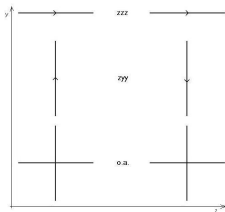
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Setup

- longitudinal plane



- transverse plane

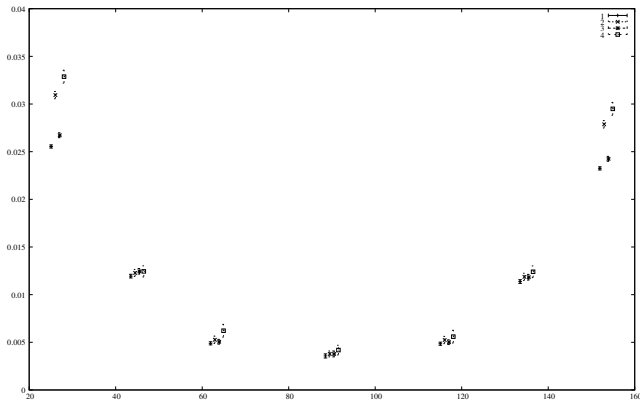


- Monte Carlo calculation of Wilson loop correlators
 - ▶ angles: $\cot \theta = 0, \pm 1/2, \pm 1, \pm 2$
 - ▶ transverse side = $1a$
 - ▶ transverse distance = $0, 1, 2a$
- Wilson action for $SU(3)$ pure gauge theory (*quenched* QCD)

$$S = \beta \sum_{n, \mu < \nu} \left\{ 1 - \frac{1}{6} \text{ReTr} U_{\mu\nu}(n) \right\}$$

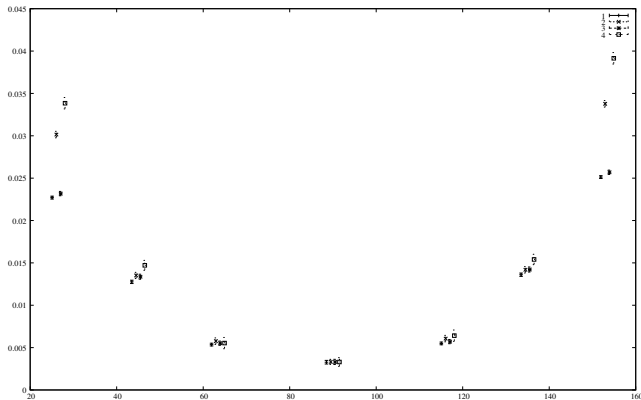
- technical details:
 - ▶ 16^4 hypercubic lattice, periodic bc
 - ▶ $\beta = 6.0 \rightarrow a \simeq 0.1 \text{ fm}$
 - ▶ 30000 measurements

Angular Dependence (zzz)



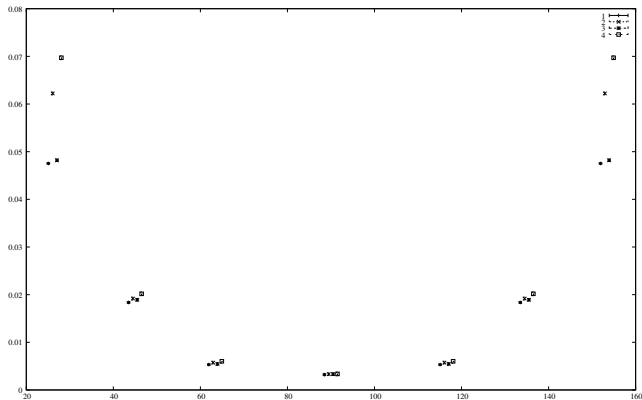
- $G_L(zzz, d=1)$ against $\theta [^\circ]$ for various L_1, L_2
- stable against variation of the loop lengths
- slower stabilisation near $\theta = 0, \pi$

Angular Dependence (zyy)



- $\mathcal{G}_L(zyy, d=1)$ against $\theta [^\circ]$ for various L_1, L_2
- stable against variation of the loop lengths
- slower stabilisation near $\theta = 0, \pi$

Angular Dependence (o.a.)



- $\mathcal{G}_L(o.a., d=1)$ against θ [°] for various L_1, L_2
- symmetric w/ respect to $\pi/2$
- sensitive only to C -even contributions

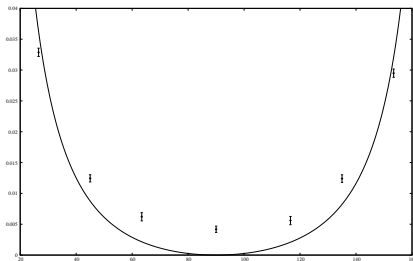
Comparison with NP Calculations

- $T \rightarrow \infty$ limit \sim largest loops available ($|\vec{l}_{||i}| \leq 8$)
 - compare data with model predictions (SVM)
 - fit data with model functions for correlators (ILM, AdS/CFT)
-
- ▶ Stochastic Vacuum Model
$$\mathcal{C}_E = \frac{2}{3} e^{-\frac{1}{3} \cot \theta K} + \frac{1}{3} e^{\frac{2}{3} \cot \theta K} - 1$$
 - ▶ Instanton Liquid Model
$$\mathcal{C}_E = l_1 \frac{1}{\sin \theta}$$
 - ▶ AdS/CFT correspondence
$$\mathcal{C}_E = A_1 \frac{1}{\sin \theta} + A_2 \cot \theta + A_3 \cos \theta \cot \theta$$

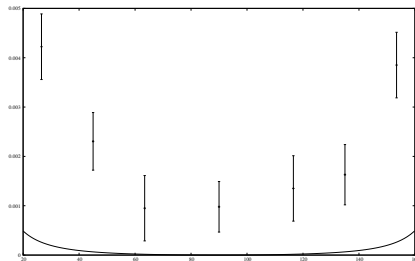
- SVM prediction

$$C_E^{\text{SVM}} = \frac{2}{3} e^{-\frac{1}{3} \cot \theta K} + \frac{1}{3} e^{\frac{2}{3} \cot \theta K} - 1$$

$$K = K(\vec{r}_{1\perp}, \vec{r}_{2\perp}, \vec{d}_\perp)$$



► zzz, $d = 1$

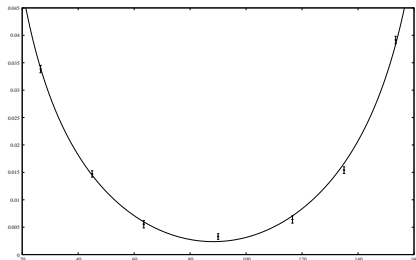


► zzz, $d = 2$

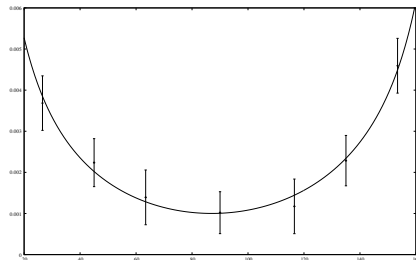
- AdS/CFT fit

$$C_E^{\text{AdS/CFT}} = A_1 \frac{1}{\sin \theta} + A_2 \cot \theta + A_3 \cos \theta \cot \theta$$

$$A_i = A_i(\vec{r}_{1\perp}, \vec{r}_{2\perp}, \vec{d}_{\perp})$$



► $z_{yy}, d = 1$

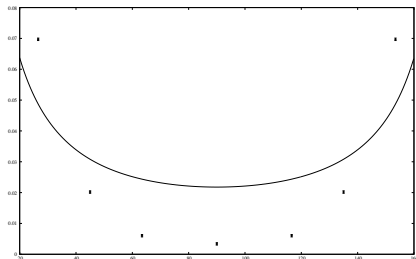


► $z_{yy}, d = 2$

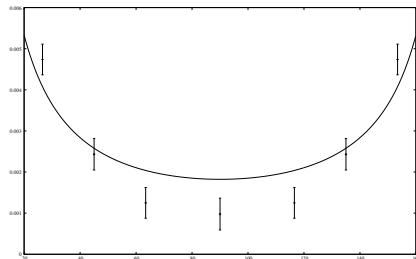
- ILM fit (1-instanton effect)

$$C_E^{\text{ILM}} = l_1 \frac{1}{\sin \theta}$$

$$l_1 = l_1(\vec{r}_{1\perp}, \vec{r}_{2\perp}, \vec{d}_{\perp})$$

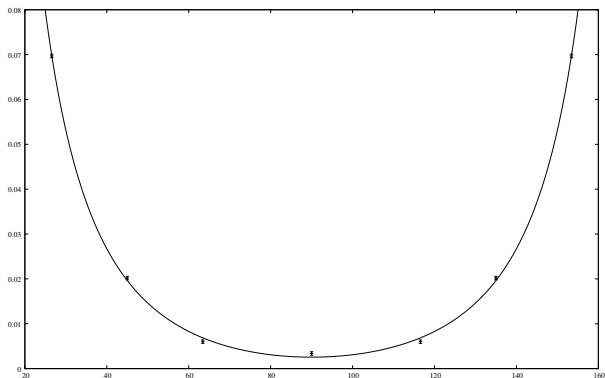


▶ o.a., $d = 1$



▶ o.a., $d = 2$

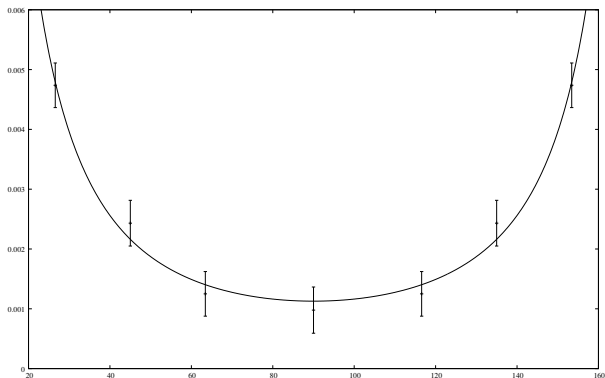
ILM + pQCD (1)



- ILM + pQCD (or 2-instanton effect)

$$C_E = C_E^{\text{ILM}} + I_2(\cot \theta)^2$$

- o.a.
- $d = 1$



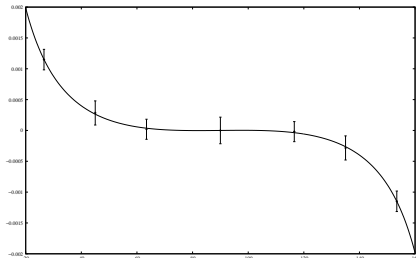
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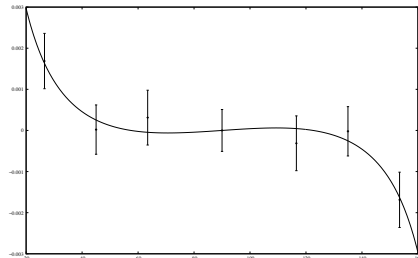
- o.a.
- $d = 2$

Odderon Contribution

- Odderon: C -odd relative of the Pomeron
- $C_E^{\text{odd}}(\theta) = \frac{1}{2}(C_E(\theta) - C_E(\pi - \theta))$
- fit with $A \cot \theta + B(\theta - \frac{\pi}{2})$



► $zzz, d = 1, L = 6$






► $zzz, d = 1, L = 8$

Conclusions and Outlook

- lattice calculations of Wilson loop correlators relevant in high-energy (soft) scattering ($\theta \rightarrow -i \log s$)
 - ▶ test existing models
 - ▶ suggest new ones?
- open problems
 - ▶ fermion effects
 - ▶ dependence of the correlator on the distance
 - ▶ dipole-dipole potential ($\theta = 0, \pi$)

References I

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