

QCD-like properties of anomalous dimensions in $\mathcal{N}=4$ SYM and ADS/CFT

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Shortly

Anomalous dimensions γ in $\mathcal{N}=4$ SYM play a crucial role in the AdS/CFT duality, and *integrability* offers new exceptional tools to calculate them at “many” loops.

In the case of the operators

$$\text{Tr}(\Phi \mathcal{D}^S \Phi \dots \Phi)$$

the $\gamma(S)$ are expected to contain important *information* encoded in their dependence on the Lorentz spin S .

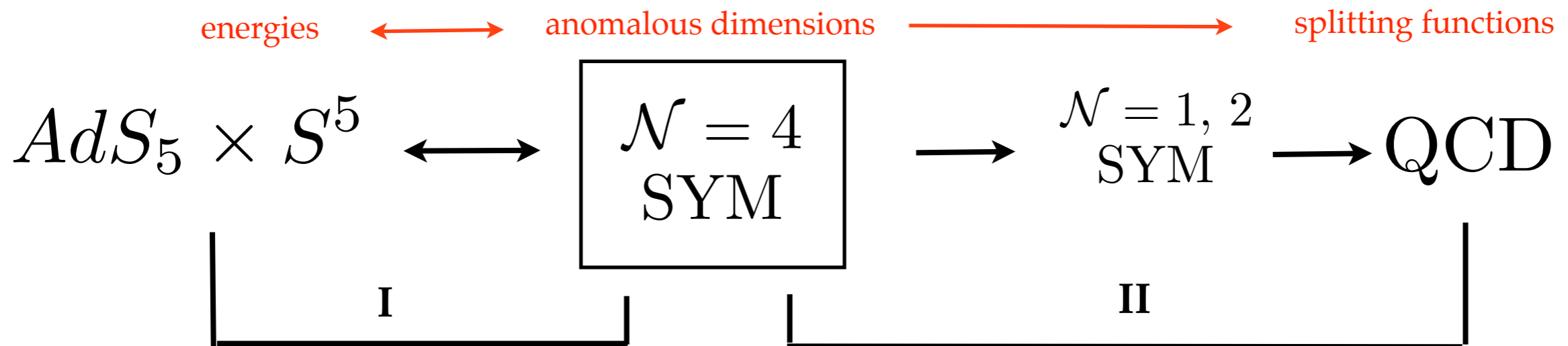
► The physical content of this information can be extracted by exploiting facts known for similar twist operators, arising in the QCD analysis of DIS.

For twist two, $\gamma(S) \leftrightarrow$ splitting functions $P(x)$.

► An analogue information is transmitted to **Strings**, and new predictions might then be available.

The central role of N=4 SYM

A triality: from string theory to strong interactions



I. AdS/CFT duality conjecture

[Maldacena, 97]

type IIB strings on $AdS_5 \times S^5 \leftrightarrow \mathcal{N}=4$ Super Yang Mills in $d=3+1$

- ▶ Agreement of the underlying symmetry supergroup $psu(2,2|4)$
- ▶ Weak/strong coupling duality $\frac{4\pi\lambda}{N} = g_s, \quad \sqrt{\lambda} = \frac{R^2}{\alpha'}, \quad (\lambda = N g_{\text{YM}}^2)$
- ▶ Prediction $E_{\text{string}} = \Delta_{\text{CFT}}$
- ▶ Planar limit $N \rightarrow \infty \Rightarrow g_s = 0$ free string. Integrability!

II. Superconformal ($\beta=0$) vs. confined ($\beta<0$), $SU(N)$ vs $SU(3)$, adjoint vs fundam.

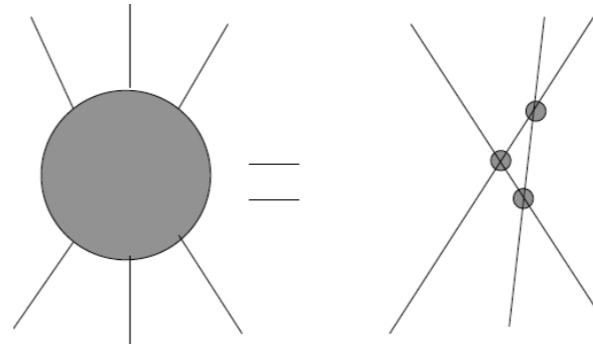
Also (III) AdS/QCD!

[Ratti, today]
[Mazzanti, tomorrow]

Integrability

Integrable structures discovered on both sides of AdS/CFT (planar limit!).

Integrable CFT: *not in the sense of factorised space-time scattering!*



Observables of the theory: correlation functions of gauge invariant local composite operators

$$\mathcal{O} = \text{Tr}(\mathcal{X}\mathcal{Y}\mathcal{Z}\mathcal{F}_{\mu\nu}\Psi(\mathcal{D}_\mu\mathcal{Z})\dots)$$

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle = \frac{1}{(x-y)^{2\Delta}}$$

scaling dimension:
classical + anomalous

$$\Delta = \Delta_0 + \gamma(\lambda)$$

It is integrable the evolution of the composite operators with the RG scale.

The planar dilatation operator \mathfrak{D} , measuring the scaling dimensions, maps to a spin chain Hamiltonian, $\mathfrak{D}(\lambda) = \mathfrak{D}_0 + \sum_{\ell \geq 1} \lambda^\ell \mathcal{H}_{integrable}^{(\ell)}$ *integrable - thus solvable* by means of a Bethe Ansatz.

Example

SU(2) operators at one loop

[Minahan, Zarembo, 02]

- ▶ Standard calculation of the two-point function
- ▶ Correspondence between operators and configurations of a spin chain

$$\text{Tr}(ZZ\Phi ZZZ\Phi\Phi\dots ZZ\Phi Z\dots) \equiv \text{Diagram}$$

➔ Dilatation operator in terms of two elementary operators acting on neighboring sites

$$\mathcal{D}^{(1)} = \frac{\lambda}{8\pi^2} \sum_{i=1}^L (1 - P_{i,i+1}) = \frac{\lambda}{8\pi^2} \mathcal{H}_{\text{XXX}}$$

Heisenberg spin chain, solvable via Bethe Ansatz.

[Bethe, 1931]

$$e^{i p_k L} = \prod_{k \neq j=1}^M S(p_k, p_j) \quad S(p_k, p_j) = \frac{u(p_i) - u(p_j) + i}{u(p_i) - u(p_j) - i}$$
$$u(p) = \frac{1}{2} \cot \frac{p}{2} \quad E = \sum_{k=1}^M 8 \sin^2 \frac{p_k}{2}$$

At three loops, Hubbard spin chain.

[Feverati, thursday]

Bethe techniques: also for *modified* N=4 SYM (e.g. orbifold)

[Beisert, Roiban 04]

[Astolfi, VF, Grignani, Semenoff 06]

Natural generalisation: **spin chain with $\mathfrak{psu}(2,2|4)$ group reps on each site.**

Solvability

Spectral problem: diagonalization of a suitable Hamiltonian.

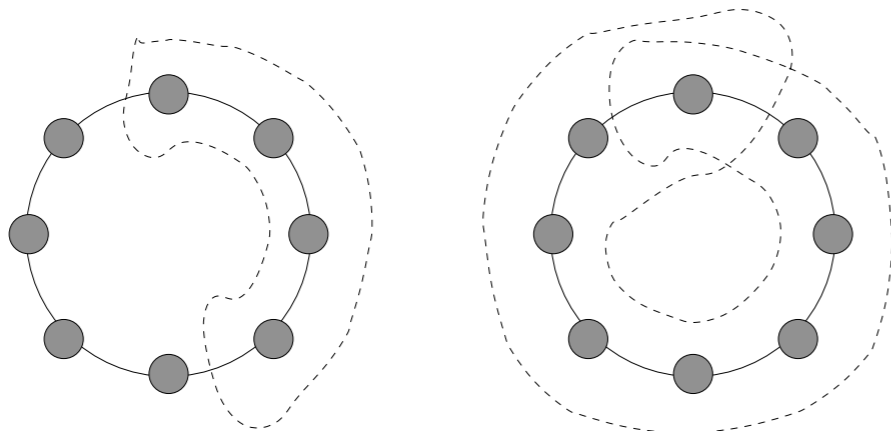
String: classical σ -model on $AdS_5 \times S^5$ also solved by a Bethe-like (integral) eq.

Easier with S-matrices (scattering of the elementary excitations on the lattice hidden inside the trace operators!) constrained by the symmetry.

► All-loop generalization: $psu(2,2|4)$ S-matrix

[Beisert, Staudacher 05]

CAVEAT: wrapping!



[Fiamberti, tomorrow]

Bethe equations are asymptotic!

Correct, for length L operators, up to $O(\lambda^L)$

Wrapping at strong coupling:
finite size correction to “giant magnons”

[Arutyunov, Frolov, Zamaklar, 06]

[Astolfi, VF, Grignani, Semenoff, 07]

$$\begin{aligned}
 1 &= \prod_{j=1}^{K_4} \frac{x_{4,j}^+}{x_{4,j}^-}, \\
 1 &= \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} + \frac{i}{2}\eta_1}{u_{1,k} - u_{2,j} - \frac{i}{2}\eta_1} \prod_{j=1}^{K_4} \frac{1 - g^2/2x_{1,k}x_{4,j}^{+\eta_1}}{1 - g^2/2x_{1,k}x_{4,j}^{-\eta_1}}, \\
 1 &= \prod_{\substack{j=1 \\ j \neq k}}^{K_2} \frac{u_{2,k} - u_{2,j} - i\eta_1}{u_{2,k} - u_{2,j} + i\eta_1} \prod_{j=1}^{K_3} \frac{u_{2,k} - u_{2,j} + \frac{i}{2}\eta_1}{u_{2,k} - u_{2,j} - \frac{i}{2}\eta_1} \prod_{j=1}^{K_1} \frac{u_{2,k} - u_{1,j} + \frac{i}{2}\eta_1}{u_{2,k} - u_{1,j} - \frac{i}{2}\eta_1}, \\
 1 &= \prod_{j=1}^{K_2} \frac{u_{3,k} - u_{2,j} + \frac{i}{2}\eta_1}{u_{3,k} - u_{2,j} - \frac{i}{2}\eta_1} \prod_{j=1}^{K_4} \frac{x_{3,k} - x_{4,j}^{+\eta_1}}{x_{3,k} - x_{4,j}^{-\eta_1}}, \\
 1 &= \left(\frac{x_{4,k}^-}{x_{4,k}^+} \right)^L \prod_{\substack{j=1 \\ j \neq k}}^{K_4} \left(\frac{x_{4,k}^{+\eta_1} - x_{4,j}^{-\eta_1}}{x_{4,k}^{-\eta_2} - x_{4,j}^{+\eta_2}} \frac{1 - g^2/2x_{4,k}^+x_{4,j}^-}{1 - g^2/2x_{4,k}^-x_{4,j}^+} \sigma^2(x_{4,k}, x_{4,j}) \right) \\
 &\quad \times \prod_{j=1}^{K_1} \frac{1 - g^2/2x_{4,k}^{-\eta_1}x_{1,j}}{1 - g^2/2x_{4,k}^{+\eta_1}x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^{-\eta_1} - x_{2,j}}{x_{4,k}^{+\eta_1} - x_{2,j}} \prod_{j=1}^{K_5} \frac{x_{4,k}^{-\eta_2} - x_{5,j}}{x_{4,k}^{+\eta_2} - x_{5,j}} \prod_{j=1}^{K_7} \frac{1 - g^2/2x_{4,k}^{-\eta_2}x_{7,j}}{1 - g^2/2x_{4,k}^{+\eta_2}x_{7,j}}, \\
 1 &= \prod_{j=1}^{K_6} \frac{u_{5,k} - u_{6,j} + \frac{i}{2}\eta_2}{u_{5,k} - u_{6,j} - \frac{i}{2}\eta_2} \prod_{j=1}^{K_4} \frac{x_{5,k} - x_{4,j}^{+\eta_2}}{x_{5,k} - x_{4,j}^{-\eta_2}}, \\
 1 &= \prod_{\substack{j=1 \\ j \neq k}}^{K_6} \frac{u_{6,k} - u_{6,j} - i\eta_2}{u_{6,k} - u_{6,j} + i\eta_2} \prod_{j=1}^{K_5} \frac{u_{6,k} - u_{5,j} + \frac{i}{2}\eta_2}{u_{6,k} - u_{5,j} - \frac{i}{2}\eta_2} \prod_{j=1}^{K_7} \frac{u_{6,k} - u_{7,j} + \frac{i}{2}\eta_2}{u_{6,k} - u_{7,j} - \frac{i}{2}\eta_2}, \\
 1 &= \prod_{j=1}^{K_6} \frac{u_{7,k} - u_{6,j} + \frac{i}{2}\eta_2}{u_{7,k} - u_{6,j} - \frac{i}{2}\eta_2} \prod_{j=1}^{K_4} \frac{1 - g^2/2x_{7,k}x_{4,j}^{+\eta_2}}{1 - g^2/2x_{7,k}x_{4,j}^{-\eta_2}}, \\
 Q_r &= \frac{1}{r-1} \sum_{j=1}^{K_4} \left(\frac{i}{(x_{4,j}^+)^{r-1}} - \frac{i}{(x_{4,j}^-)^{r-1}} \right), \quad \delta D = g^2 Q_2 = g^2 \sum_{j=1}^{K_4} \left(\frac{i}{x_{4,j}^+} - \frac{i}{x_{4,j}^-} \right).
 \end{aligned}$$

Remarkable outcomes

The large spin limit

Spectacular interpolation weak/strong coupling: the scaling function/ cusp anomaly

- ▶ QCD: logarithmic scaling in leading twist operators at large spin S ($x \rightarrow 1$)

$$\gamma(S) = 2 \Gamma_{\text{cusp}}(\alpha) \log S + \mathcal{O}(S^0) \qquad \mathcal{O}_S = \bar{q}(\gamma_+ \mathcal{D}^+)^S q$$

Γ_{cusp} governs the renormalization of a Wilson loops evaluated over a closed contour with a cusp.

- ▶ SYM: Twist two operators in $\mathfrak{sl}(2) \subset \mathfrak{psu}(2,2|4)$ via Bethe Ansatz [Bombardelli, tomorrow]

$$\gamma(S) = f(\lambda) \log S + \mathcal{O}(S^0) \qquad \mathcal{O}_S = \text{Tr}(\varphi \mathcal{D}_+^S \varphi)$$

- At weak coupling $f(\lambda) = \frac{\lambda}{2\pi^2} - \frac{\lambda^2}{96\pi^2} + \dots$

- ✓ Agreement with MHV n-point gluon amplitudes of N=4 SYM at four loops !

[Bern, Czakon, Dixon, Kosower, Smirnov, 06]

- At strong coupling $f(\lambda) = \frac{\sqrt{\lambda}}{2\pi} - \frac{3 \log 2}{2\pi} + \dots$

numerically
analytically

[Benvenuti et al 07, Alday et al 07, Kostov et al 07, Beccaria VF 0703]

[Basso, Korchemsky 07]

- ✓ Agreement with two loops string calculations !
and *predictions* up to 40 loops..

[Roiban, Tseytlin 07]

Remarkable “byproducts”

- ▶ Wilson loops vs gluon scattering amplitudes at weak and strong coupling
[Alday, Maldacena 07]
[Korchemsky et al. 07, Bern et al. 08, ...]
- ▶ Generalised scaling function $f(\lambda, J)$
[Freyult Rej Staudacher 07, Roiban Tseytlin 07,
Gromov 08, Fioravanti et al. 1& 2 08]

and natural outlook!

**How the *triality* survives in the large spin expansion
beyond the leading order ?**

- ▶ Sectors other than $sl(2)$?
- ▶ Logarithmic scaling also at higher twist?
- ▶ Predictions for strings?

N=4 SYM and QCD interplay: I

QCD-inspired closed formulas

Crucial to explore the different regimes $x \rightarrow 0$, $x \rightarrow 1$.

Bethe eqs: highly non linear, complicated. Appropriate Ansatz for coefficients needed!

[e.g. Operators with variable length $\text{Tr } F^L$]

[Beccaria Forini, 0710]

Maximum transcendentality principle, KLOV [Kotikov, Lipatov, Onishchenko, Velizhanin, 04]

The N=4 universal twist two anomalous dimension at n loops is a linear combination of harmonic sums of transcendentality $2n - 1$.

3-loop $\gamma(S)_{\text{uni}}$ extracted from the “most transcendental terms” of the QCD result.

[Moch, Vermaseren, Vogt, 04]

✓ Applied to the numbers coming from the Bethe eqs it works!

$$\gamma^{(1)}(S) = \sum_{|\tau|=1} c_{\tau} S_{\tau}(S) = c S_1(S)$$

$$\gamma^{(2)}(S) = \sum_{|\tau|=3} c_{\tau} S_{\tau}(S)$$

$$\gamma^{(3)}(S) = \sum_{|\tau|=5} c_{\tau} S_{\tau}(S)$$

(Nested) harmonic sums

$$S_a(S) = \sum_{n=1}^S \frac{(\text{sign}(a))^n}{n^{|a|}}$$

$$S_{a,b}(S) = \sum_{n=1}^S \frac{(\text{sign}(a))^n}{n^{|a|}} S_b(n)$$

Is it universal in twist (also twist 3)? and in flavor (also gauginos and gluons)?

Generalising KLOV:

► Twist-3 $\mathfrak{sl}(2)$, three scalars $\mathcal{O}_S = \text{Tr} \mathcal{D}_+^S(\varphi)^3$ [Beccaria 07]

Four-loop closed formula for $\gamma(S)$ in terms of $S_{a_1 \dots a_n}(S/2)$ [Kotikov et al 07]

► Twist-3 $\mathfrak{sl}(2|1)$, three gauginos $\mathcal{O}_S = \text{Tr} \mathcal{D}_+^S(\lambda)^3$ [Beccaria 07]

Theorem $\gamma^{\lambda\lambda\lambda}(S) = \gamma_{\text{twist-2}}(S+2)$

► The most *complete* (and complicated) test of universality is the case of three gauge fields, closed only at one loop \rightarrow representing the full $\mathfrak{psu}(2,2|4)$.

$$\mathcal{O}_S = \text{Tr} \mathcal{D}_+^S(A)^3$$

[Beccaria 07]
[Beccaria, Forini 08]

- Identify the superconformal primary state to which apply the Bethe eqs.
- Violation/generalization of the KLOV principle.

$$\gamma_n = \sum_{\tau=0}^{2n-1} \gamma_n^{(\tau)}, \quad \gamma_n^{(\tau)} = \sum_{k+l=\tau} \frac{\mathcal{H}_{\tau, \ell}(n)}{(n+1)^k}$$

Up to four loops!

$$\gamma_1 = 4S_1 + \frac{2}{n+1} + 4,$$

$$\gamma_2 = -2S_3 - 4S_1S_2 - \frac{2S_2}{n+1} - \frac{2S_1}{(n+1)^2} - \frac{2}{(n+1)^3} - 4S_2 - \frac{2}{(n+1)^2} - 8,$$

Harmonic sums evaluated in $S/2+1$.

N=4 SYM and QCD interplay: II

$x \rightarrow 0$, *negative spin* S (BFKL equation)

The (analytically continued) anomalous dimension for twist two $sl(2)$ operators *maximally violates, at four loops*, the BFKL prediction for the (Regge) poles

→ Breakdown of the Bethe eqs. at four loops

[Lipatov, Staudacher et al. 07]

N=4 SYM and QCD interplay: III

$x \rightarrow 1$, *large spin* S (quasi elastic limit)

► *Leading* logarithmic behavior: *universal in twist and flavors for all gauge theories!*

$$\gamma(S) = 2 \Gamma_{\text{cusp}}(\alpha) \log S + \mathcal{O}(S^0)$$

[Belitsky, Gorsky, Korchemsky, 06]

► *Subleading* logarithmic behavior: much less clear!

■ MVV relations for twist two ops

[Moch, Vermaseren, Vogt, 04]

$$\gamma_{\sigma}(S) = A \log \bar{S} + B + C_{\sigma} \frac{\log \bar{S}}{S}$$

$$C_{\sigma} = -\frac{1}{2} \sigma A^2$$

Highly non trivial constraints, since A, B, C are functions of the coupling.

Structural explanation: **revisiting the parton evolution** in the $x \rightarrow 1$ regime

Anomalous dimensions vs splitting functions

- ▶ DGLAP evolution equations for f_σ , parton distribution function (DIS) or fragmentation functions (e+e- annihilation into hadrons)

$$\frac{df_\sigma(x, Q^2)}{d \log Q^2} = \int_x^1 \frac{dz}{z} P_\sigma(z, \alpha_s) f_\sigma\left(\frac{x}{z}, Q^2\right)$$

$$\frac{df_\sigma(S, Q^2)}{d \log Q^2} = \gamma_\sigma(S) f_\sigma(S, Q^2) \quad \gamma_\sigma(S) = \int_0^1 dx x^{N-1} P_\sigma(x)$$

The Mellin transforms of the space or time-like ($\sigma = \pm 1$) splitting function P_σ are anomalous dimensions \rightarrow in the space-like case, those of twist-2 operators.

- ▶ Proposal by Gribov-Lipatov: the two splitting functions *coincide* and

$$P(x) = -x P\left(\frac{1}{x}\right)$$

GL reciprocity [Gribov, Lipatov, 72]

Broken in QCD beyond 1-loop

[Curci, Furmanskyy, Petronzio, 80]

- ▶ Proposal by Marchesini-Dokshitzer: Reciprocity Respecting Evolution Equations

$$\frac{df_\sigma(x, Q^2)}{d \log Q^2} = \int_x^1 \frac{dz}{z} \mathcal{P}(z, \alpha_s) f_\sigma\left(\frac{x}{z}, z^\sigma Q^2\right)$$

RREE

with a universal kernel \mathcal{P} .

[Marchesini, Dokshitzer, Salam, 05]

- ✓ Verified at three loops for non singlet twist two

[Mitov, Moch, Vogt 06]

RREE are solved by the non-linear relation

$$\gamma_\sigma = \mathcal{P} \left(S - \frac{1}{2} \sigma \gamma_\sigma(S) \right)$$

■ If the kernel satisfies the GL reciprocity

$$\mathcal{P}(x) = -x \mathcal{P} \left(\frac{1}{x} \right) \quad \text{or} \quad \mathcal{P}(S) = f(S(S+1))$$

✓ MVV relations for subleading terms are satisfied !

Generalized Reciprocity in N=4 SYM

The function \mathcal{P} defined by $\gamma = \mathcal{P} \left(S + \frac{1}{2} \gamma(S) \right)$

is a reciprocity respecting (RR) kernel if its *large spin* expansion takes the form

$$\mathcal{P}(S) = \sum_n \frac{a_n (\log J)}{J^{2n}} \quad J^2 = (S + L s)(S + L s - 1)$$

L: twist s: collinear spin =

φ	λ	A
$\frac{1}{2}$	1	$\frac{3}{2}$

[Basso, Korchemsky 06]

Checks of Reciprocity

- ▶ Step 1: solve for

$$\mathcal{P}(S) = \sum_{k=1}^{\infty} \left(-\frac{1}{2} \partial_S \right)^{k-1} [\gamma(S)]^k = \gamma - \frac{1}{4} (\gamma^2)' + \frac{1}{24} (\gamma^3)'' + \dots$$

As the anomalous dimension, P will be a perturbative series (weak coupling)

- ▶ Step 2: Expand in large S , and check the parity invariance wrt J^2

Ex. twist two

$$\mathcal{P}_1 = 8 \ln J + \frac{4}{3} \frac{1}{J^2} - \frac{4}{15} \frac{1}{J^4},$$

$$\mathcal{P}_2 = -\frac{8}{3} \pi^2 \ln J - 24 \zeta_3 + \left(8 - \frac{4\pi^2}{9} \right) \frac{1}{J^2} + \left(4 + \frac{4\pi^2}{45} \right) \frac{1}{J^4}$$

✓ All γ_S for twist-2 in QCD

[Basso, Korchemsky 06]

✓ Universal twist-2 in $\mathcal{N}=4$ SYM

[Basso, Korchemsky 06]

✓ Twist-3 scalars, gauginos, gluons (in closed form!) in $\mathcal{N}=4$ SYM

[Beccaria, Marchesini, Dokshitzer 07] [Beccaria 07] [Beccaria Forini 08]

It is an (empirical) evidence, but *what is its origin?*

Reciprocity and AdS/CFT

Anomalous dimensions of operators in N=4 SYM



Energies of semiclassical strings in $AdS_5 \times S^5$

Classical sols of the $AdS_5 \times S^5$ σ -model: states belonging to reprs of $SO(2, 4) \times SO(6)$.

➔ Classified by 6 charges $(E, S_1, S_2; J_1, J_2, J_3)$.

► Operators with large Lorentz spin correspond to *folded* strings rotating in $AdS_3 \times S^1$ and the *classical* result for the energy reproduces the logarithmic behavior!

$$E = S + \frac{\sqrt{\lambda}}{\pi} \log \frac{S}{\sqrt{\lambda}} + \dots \quad S \gg \sqrt{\lambda} \quad \text{[Gubser, Klebanov, Polyakov 02]}$$

i.e. the behavior of the (classical + anomalous) dimension of twist two operators.

► The energy organizes in the semiclassical expansion

$$E = \sqrt{\lambda} \left[E_0 + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{(\sqrt{\lambda})^2} + \dots \right]$$

and its relation to the anomalous dimension is (supposed to be) $\gamma(S) = E - S - L$

The anomalous dimensions inherits the semiclassical expansion!

- Cusp anomaly ($\log \mathcal{S}$): prediction from Bethe Eqs up to 40 quantum corrections!

$$f(\lambda) = \sqrt{\lambda} \left[\frac{1}{\pi} - \frac{3 \log 2}{\pi \sqrt{\lambda}} - \frac{K}{(\sqrt{\lambda})^2} - \left(\frac{27}{32} \zeta_3 + 3 K \log 2 \right) \frac{1}{(\sqrt{\lambda})^3} + \dots \right]$$

And the subleading terms?

► Step 1. Reciprocity has to be rewritten $\mathcal{P}(\mathcal{S}) = \sqrt{\lambda} \left[\mathcal{P}_0(\mathcal{S}) + \frac{\mathcal{P}_1(\mathcal{S})}{\sqrt{\lambda}} + \frac{\mathcal{P}_2(\mathcal{S})}{(\sqrt{\lambda})^2} + \dots \right]$

- Step 2. Reciprocity has to be checked!

✓ Folded string for large twist L (only classical energy)

[Basso, Korchemsky 06]

✓ Folded string for finite (small) twist

[work in progress]

- Step 3. Reciprocity should give *new predictions for loop corrections!*

[work in progress]

$$\begin{aligned} E - S = \gamma(\mathcal{S}) &= f(\lambda) \log \mathcal{S} + f_1(\lambda) \frac{\log \mathcal{S}}{\mathcal{S}} + \dots \\ &= \sqrt{\lambda} \left(a_0 + \frac{b_0}{\sqrt{\lambda}} + \frac{c_0}{(\sqrt{\lambda})^2} + \dots \right) \log \mathcal{S} + \lambda \left(a_1 + \frac{b_1}{\sqrt{\lambda}} + \frac{c_1}{(\sqrt{\lambda})^2} + \dots \right) \frac{\log \mathcal{S}}{\mathcal{S}} + \dots \end{aligned}$$

$$b_1 = -\frac{3 \log 2}{\pi^2 \sqrt{\lambda}}, \quad c_1 = -\frac{K}{\pi} + \frac{9 \log^2 2}{2\pi^2}$$

Conclusions

CFT/

- ▶ Long range Bethe equations \rightarrow multiloop anomalous dimensions.
- ▶ Closed formulas *QCD-inspired*
- ▶ Physical properties: BFKL singularities, reciprocity. Why?

/AdS

- ▶ Anomalous dimensions at strong coupling
- ▶ Reciprocity in AdS. Why?
- ▶ Predictions!

QCD

- ▶ Complete solution of $N = 4$ SYM should provide a “one-line-all-orders description” of the *major* part of QCD parton dynamics.

Outlook 1: The Konishi anomalous dimension

[Fiamberti, tomorrow]

$$\gamma = 4 + 12 \hat{\lambda} - 48 \hat{\lambda}^2 + 336 \hat{\lambda}^3 + \gamma_4 \hat{\lambda}^4 + \dots$$

On the four loop contribution, there are many predictions

$$\text{E.g. } \gamma_4 = -\frac{5307}{2} - 564\zeta_3$$

[Lipatov, Staudacher et al. 07]

and two different results from a field theoretical calculation

$$\gamma_4 = -2584 + 384 \zeta_3 - 1440 \zeta_5$$

[Zanon et al. 07]

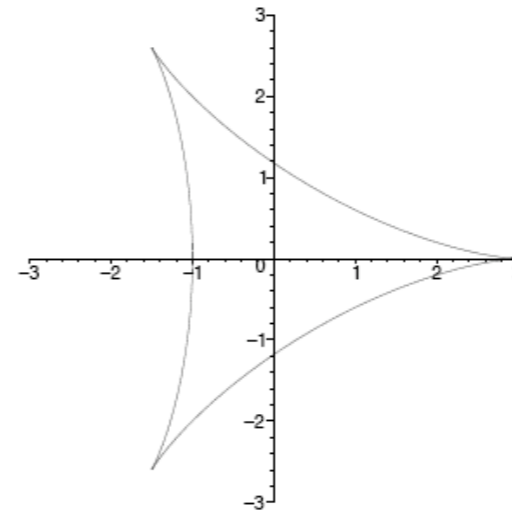
$$\gamma_4 = -2607 - 28 \zeta_3 - 140 \zeta_5$$

[Mann Keeler 08]

Reciprocity (and **BFKL**) might give some hints.

Outlook 2: Spiky strings vs. higher twist

Proper correspondent to twist L operators
are string configurations with L spikes



[Kruczenski. 04]

[Kruczenski, Tseytlin 08]

Outlook 3: Hints for other universalities

In the large S expansion, the coefficients of the terms $\ln^m S/S^m$ look *independent on the flavor and on the twist*, only partially due to reciprocity.